PROBLEM SET 9 NE290H BARNARD & LUND

30 POINTS

PROBLEM 1

a) SHOW THAT $r_z^2 = C\langle z^2\rangle$ and find C, for a Neuffer distribution

$$f(z,z',s) = \frac{3N}{2\pi\varepsilon_z}\sqrt{1-\frac{z^2}{r_z^2}-\frac{r_z^2(z'-r_z'z/r_z)^2}{\varepsilon_z^2}}$$

for $-r_z < z < r_z$

& $\frac{r_z'z}{r_z} - \frac{\varepsilon_z}{r_z}\sqrt{1-\frac{z^2}{r_z^2}} < z' < \frac{r_z'z}{r_z} + \frac{\varepsilon_z}{r_z}\sqrt{1-\frac{z^2}{r_z^2}}$

You may use the result given in class that:

$$\lambda(z) \equiv q\int_{-\infty}^{\infty} f(z,z')\,dz' = \frac{3}{4}\frac{qN}{r_z}\left(1-\frac{z^2}{r_z^2}\right) \quad \text{for } -r_z < z < r_z$$

Here N is the total number of particles in the bunch, & $\varepsilon_z^2 = 25\left[\langle z^2\rangle\langle z'^2\rangle - \langle zz'\rangle^2\right]$ = longitudinal emittance, and q is the ion charge.

b). Calculate $\langle z^4\rangle$ for a NEUFFER DISTRIBUTION

c). FOR THE CASE $r_z' = 0$, WHAT IS $\langle zz'\rangle$?
FOR THE CASE $r_z' = 0$, WHAT IS $\langle z'^2\rangle$?
You may appeal to symmetry and all results listed in the statement of the problem.

30 POINTS

PROBLEM 2: WHEN LONGITUDINAL EMITTANCE IS INCLUDED IN THE NON-RELATIVISTIC LONGITUDINAL ENVELOPE EQUATION, DESCRIBING THE LENGTH L OF A PULSE WITH PARABOLIC LINE CHARGE DENSITY UNDERGOING BUNCH COMPRESSION,

$$\frac{d^2L}{ds^2} = \frac{16\,\varepsilon_z^2}{L^3} + \frac{12\,g\,q\,Q_c}{4\pi\varepsilon_0\, m v^2 L^2}$$

where Q_c is the total charge in the bunch,

$\varepsilon_z^2 \equiv$ longitudinal emittance $= 25\left[\langle z^2\rangle\langle z'^2\rangle - \langle z z'\rangle^2\right]$.

SHOW THAT THE INITIAL VELOCITY TILT $\frac{\Delta V}{V_0}$ REQUIRED TO COMPRESS THE BEAM TO "STAGNATION" [i.e. to the point where $\frac{dL}{ds} = 0$) IS GIVEN BY:

$$\frac{\Delta V^2}{V_0^2} = \frac{16\,\varepsilon_z^2}{L_0^2}\left[C^2 - 1\right] + \frac{24\,g\,q\,Q_c}{4\pi\varepsilon_0 L_0 m v_0^2}\left[C - 1\right]$$

WHERE $L_0 = L$ at $s = 0$,

$L_f = L$ at the stagnation point

& $C \equiv L_0 / L_f$ = compression ratio > 1

& $\frac{\Delta V}{V_0} = -L_0' = \left.\frac{dL}{ds}\right|_{s=0}$ $\quad V_0$ = longitudinal velocity of beam center