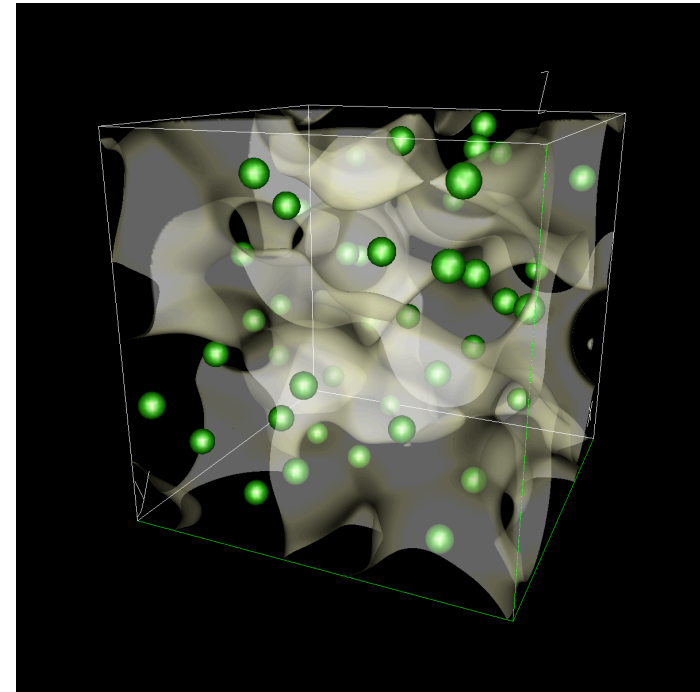
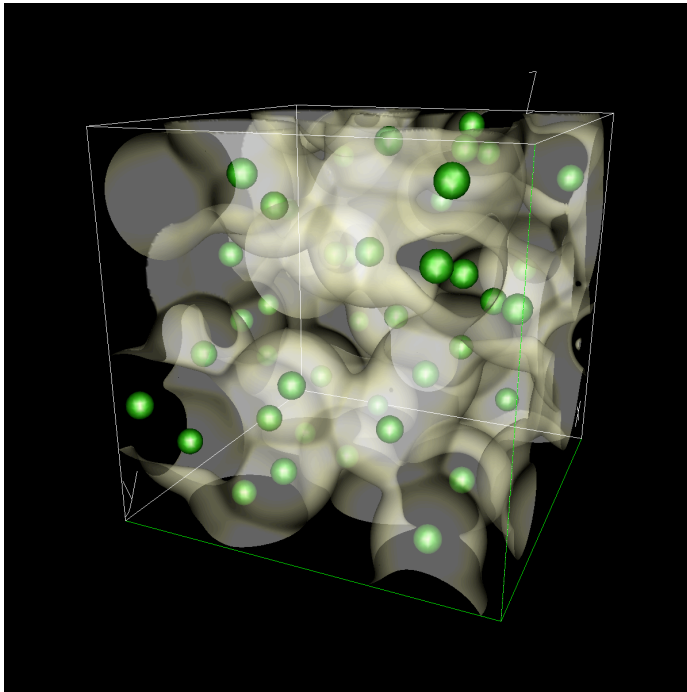


# QMD Simulations of Warm Dense Matter in support of High Energy Density Physics Experiments on the Z Machine



**Mike Desjarlais**  
**HEDP Theory &**  
**ICF Target Design**  
**Sandia National Laboratories**

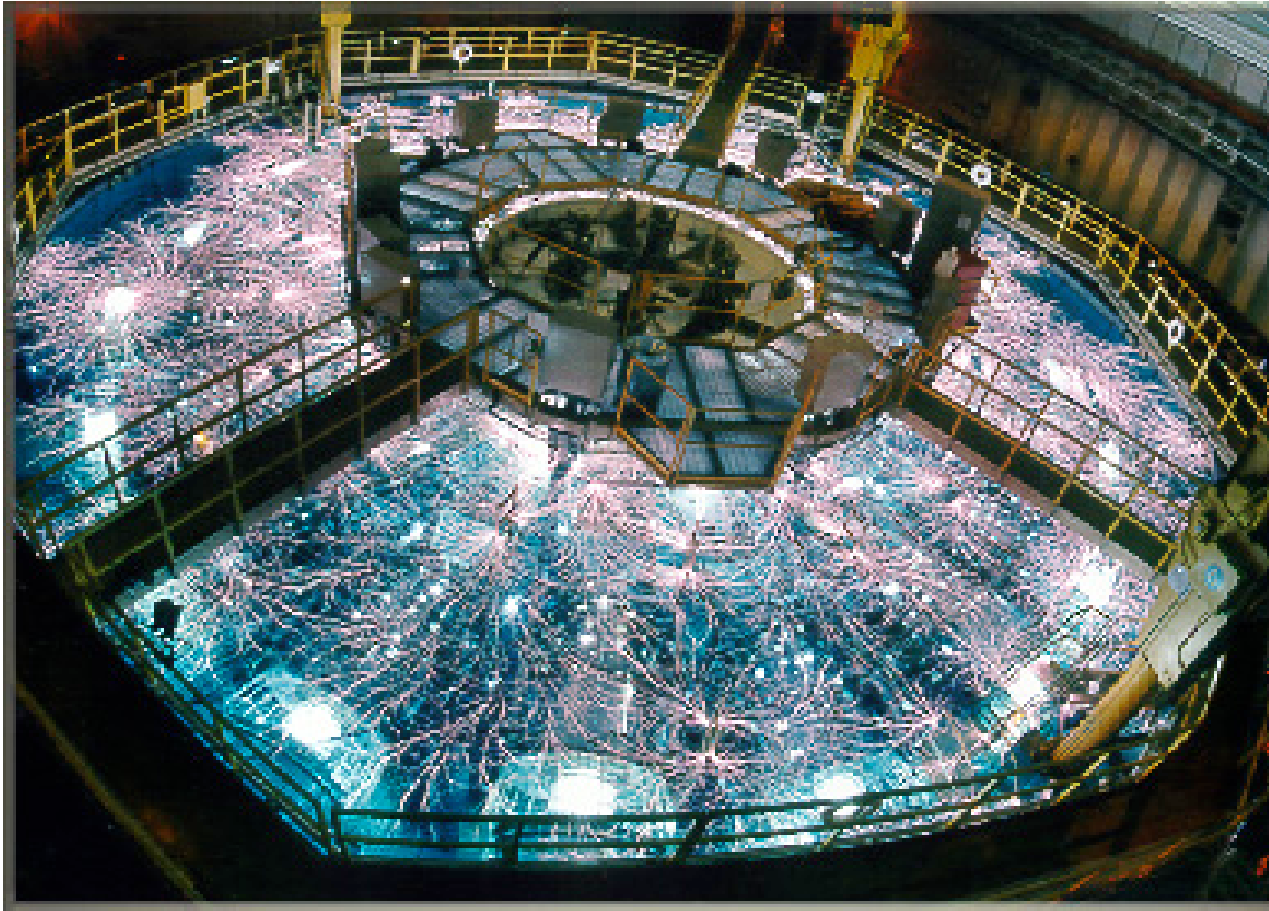


**Workshop on**  
**Accelerator-Driven**  
**Warm Dense Matter**  
**February 22-24, 2006**



## **Sandia's Z Machine is used for several HEDP experimental campaigns**

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**Z pinches for Inertial Confinement Fusion**

**Magnetically launched flyer plates for EOS studies**

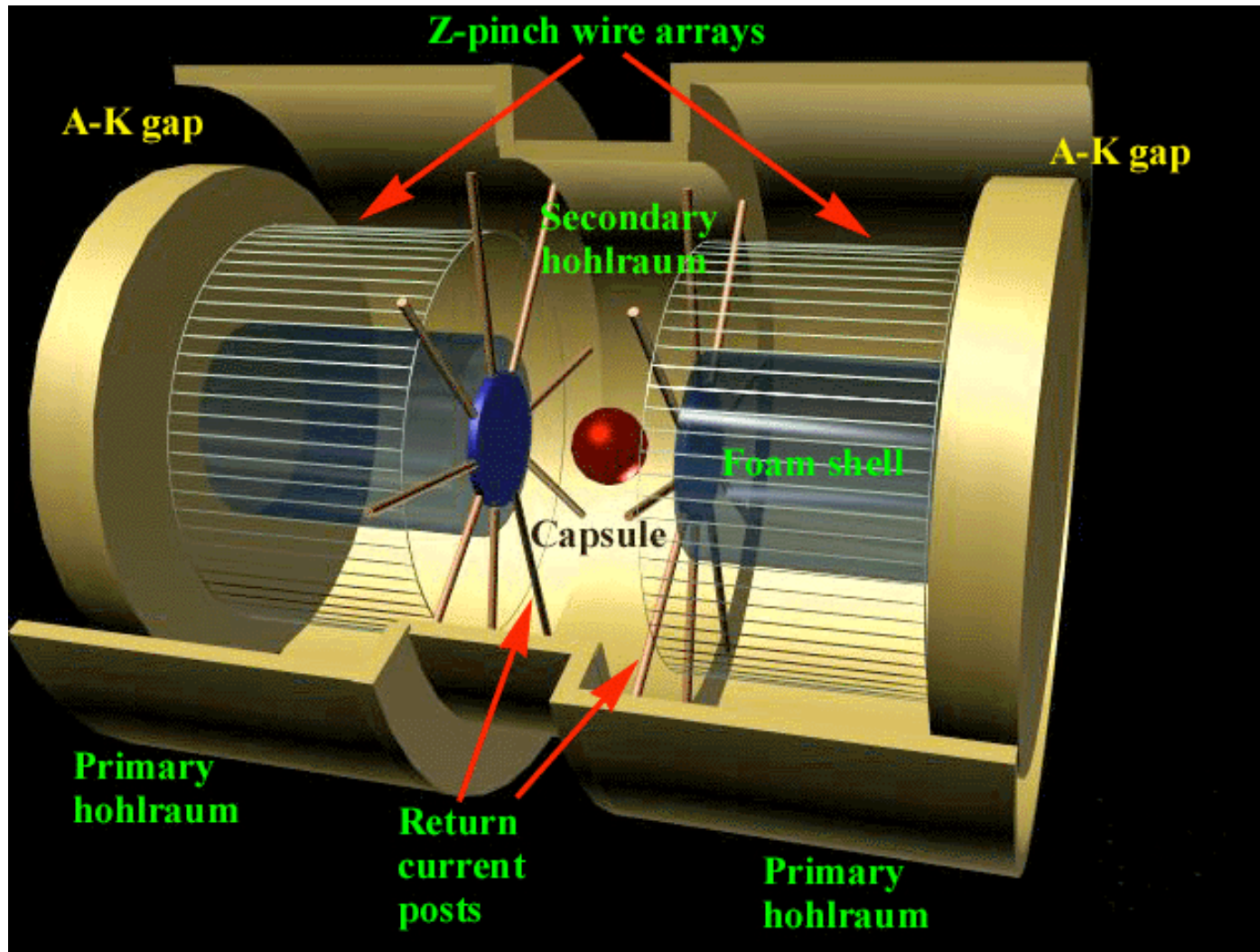
**Isentropic compression for EOS studies**

**The Z machine delivers more than 20 million Amperes of current  
ZR will generate more than 25**



**We routinely perform large scale computer simulations to model the complex geometries and physics in our HEDP experiments**

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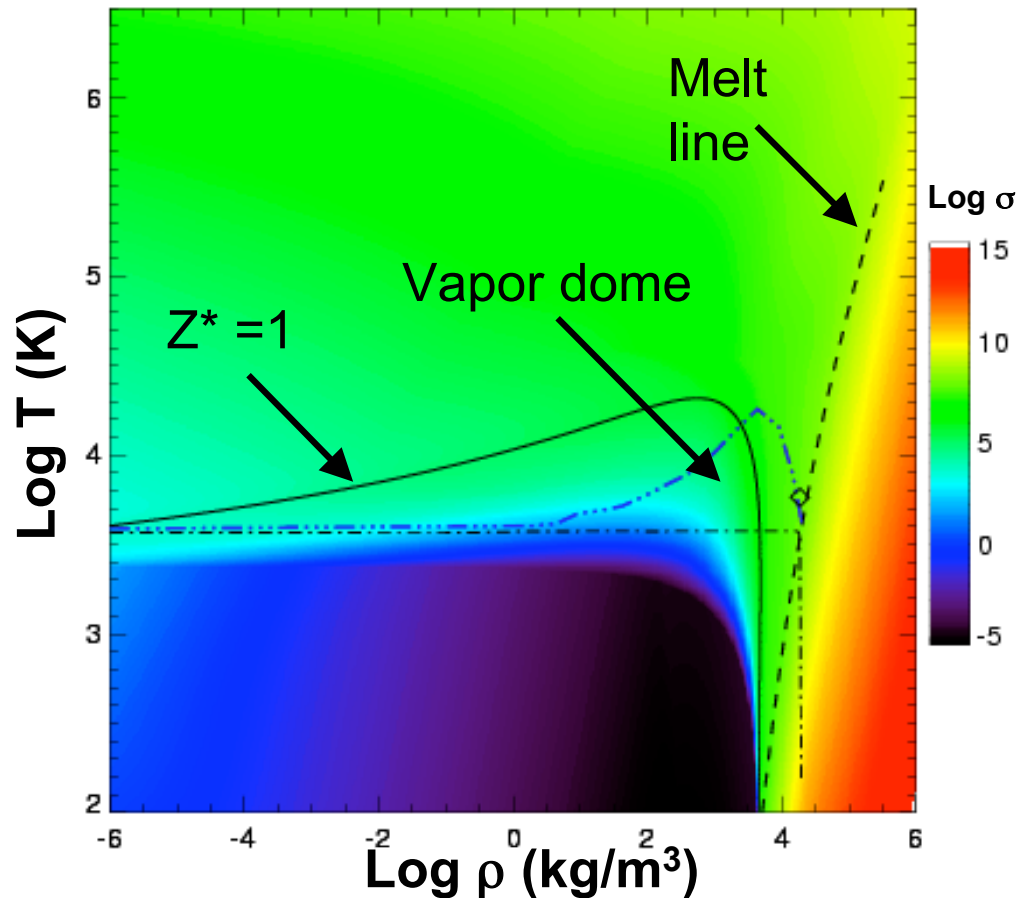


This is a double-ended Z pinch with an ICF target in the center



## HEDP computer simulations rely on “physics packages”: Conductivities, Equations of State, and Opacities

### Tungsten Conductivity



Definitions of Warm Dense Matter are varied, but generally center around **strongly coupled ions and moderately degenerate electrons** --- many different interactions are comparable.

A **quantum mechanical** treatment is generally necessary.

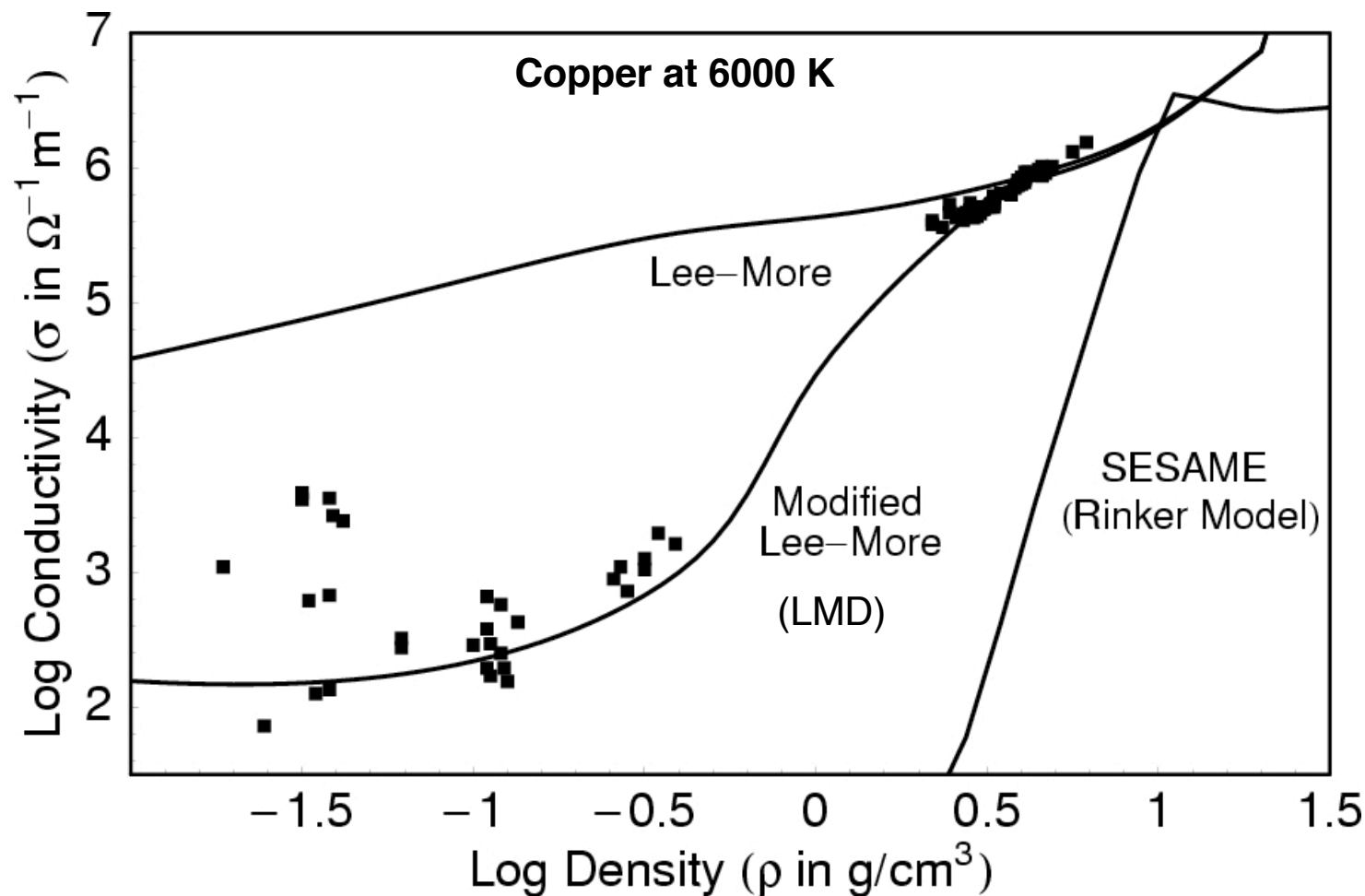
Most wide-range equations of state interpolate through this difficult area.

This highly structured portion of phase space is  
Warm Dense Matter

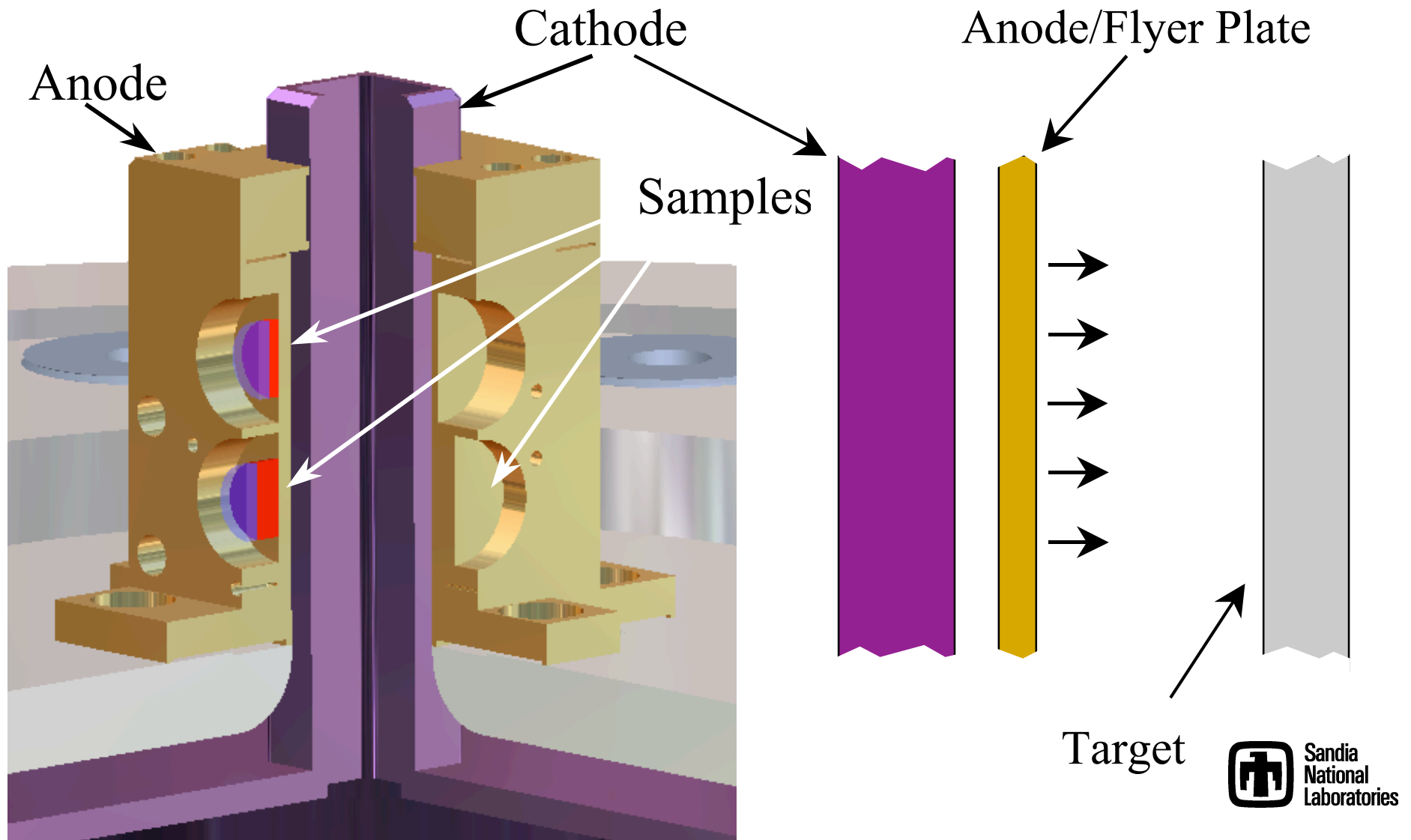


## Modifications of the Lee-More algorithm were made to obtain an improved wide-range model\*

\* M. P. Desjarlais, Contrib. Plasma Phys. **41** (2001) 2-3, 267-270



**A demanding application: Ultra-high velocity magnetically launched flyer plates (30 km/sec, Multi-Mbar pressures)**

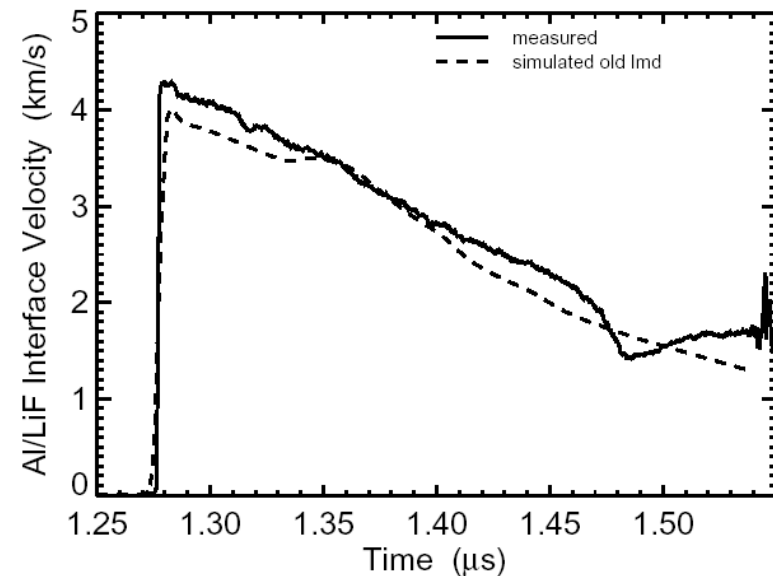
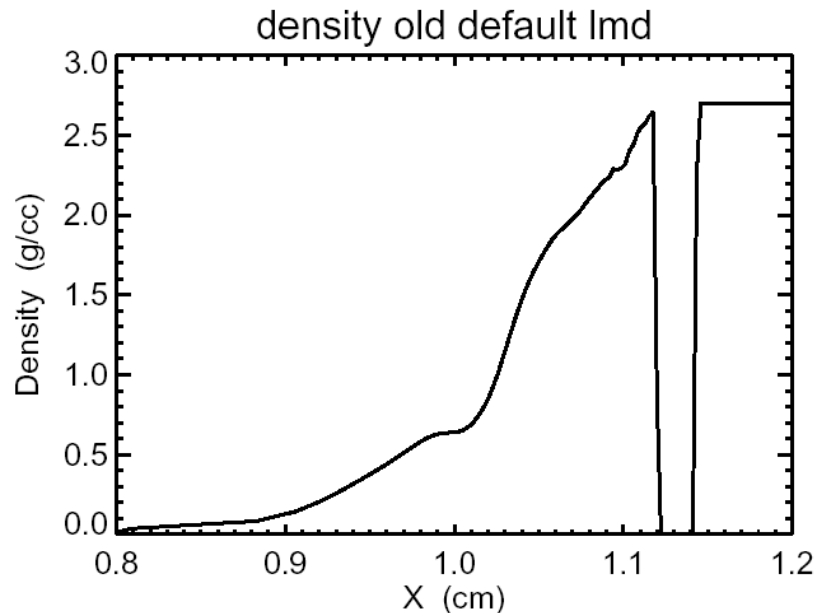




## We simulated these magnetically launched flyer plates using the modified Lee-More (LMD) conductivities

Detailed comparison between simulations and experiments for magnetically launched flyer plates suggested that our *improved* conductivities were still not sufficiently accurate for the warm dense liquid aluminum.

Simulations by Ray Lemke with Sandia's 3D Rad-MHD ALEGRA code



For many of our applications we require conductivities accurate to well within a factor of two.



## **We are using Density Functional Theory (DFT) to perform Quantum Molecular Dynamics (QMD) simulations of Warm Dense Matter**

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- The simulations are performed with VASP (Vienna Ab initio Simulation Program), a plane wave density functional code
- Exchange and Correlation functionals are Generalized Gradient Approximation (GGA/PBE)
- We typically use up to 256 atoms, but it varies depending on density and the number of electrons we need to carry (the valence); We use Projector Augmented Wave (PAW) all-electron, frozen core potentials for the atoms
- We generally perform our simulations in the Canonical Ensemble (N,V,T) using either velocity scaling or a Nosé-Hoover thermostat to regulate the temperature; Fermi statistics for the electrons
- Typical runs cover one to twenty picoseconds



## The Kubo-Greenwood formula is used to calculate the frequency dependent electrical conductivity

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$$\sigma_{\mathbf{k}}(\omega) = \frac{2\pi e^2 \hbar^2}{3m^2 \omega \Omega} \sum_{\alpha=1}^3 \sum_{j=1}^N \sum_{i=1}^N (F(\varepsilon_{i,\mathbf{k}}) - F(\varepsilon_{j,\mathbf{k}})) \left| \langle \Psi_{j,\mathbf{k}} | \nabla_{\alpha} | \Psi_{i,\mathbf{k}} \rangle \right|^2 \delta(\varepsilon_{j,\mathbf{k}} - \varepsilon_{i,\mathbf{k}} - \hbar\omega),$$

where  $e$  and  $m$  are the electron charge and mass. The  $i$  and  $j$  summations are over the  $N$  discrete bands of the triply periodic calculation for the cubic supercell with volume  $\Omega$ . The coordinate index is  $\alpha$  and in general we average over  $\alpha$  to improve the statistics.  $F(\varepsilon_{i,\mathbf{k}})$  is the Fermi weight corresponding to the energy for the  $i$ -th band at  $\mathbf{k}$  with wavefunction  $\Psi_{i,\mathbf{k}}$ .

We integrate over the Brillouin zone using the method of special  $\mathbf{k}$ -points

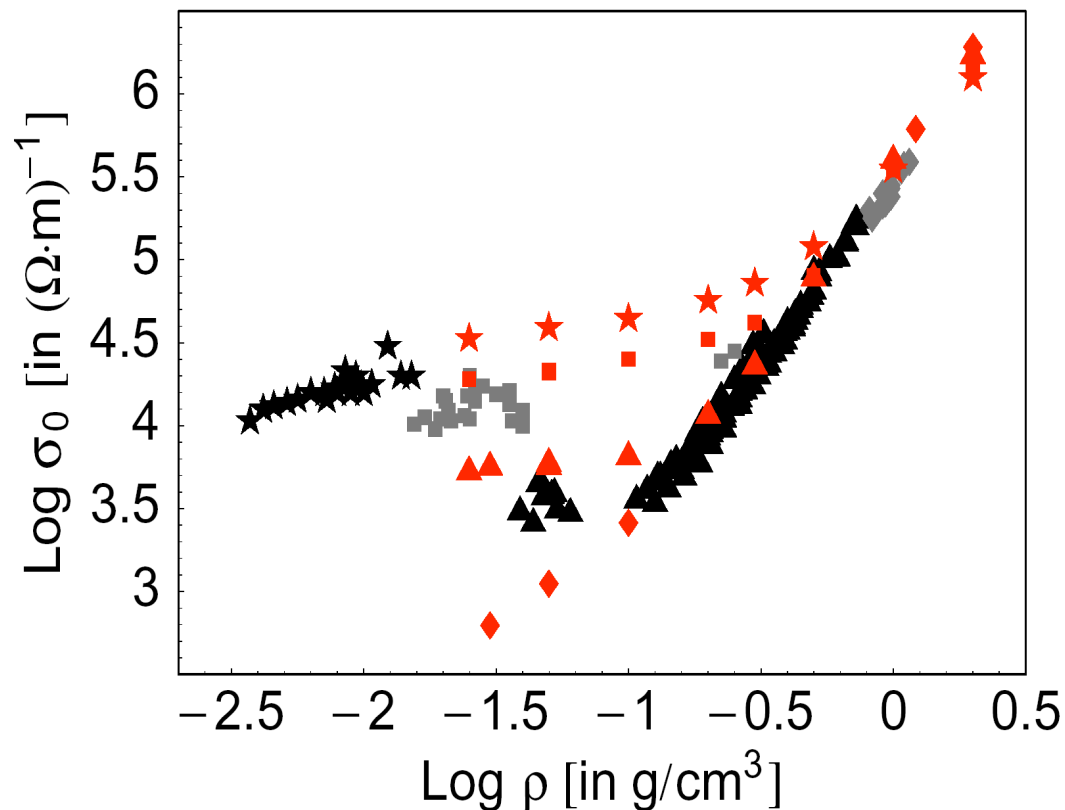
$$\sigma(\omega) = \sum_{\mathbf{k}} \sigma_{\mathbf{k}}(\omega) W(\mathbf{k}) ,$$

and average over 10 to 20 configurations selected from the MD run.

This is really nothing more than the quantum analog of the classical current-current correlation function representation of the conductivity



The QMD-KG results are in good agreement with DeSilva's data over a two decade range of density



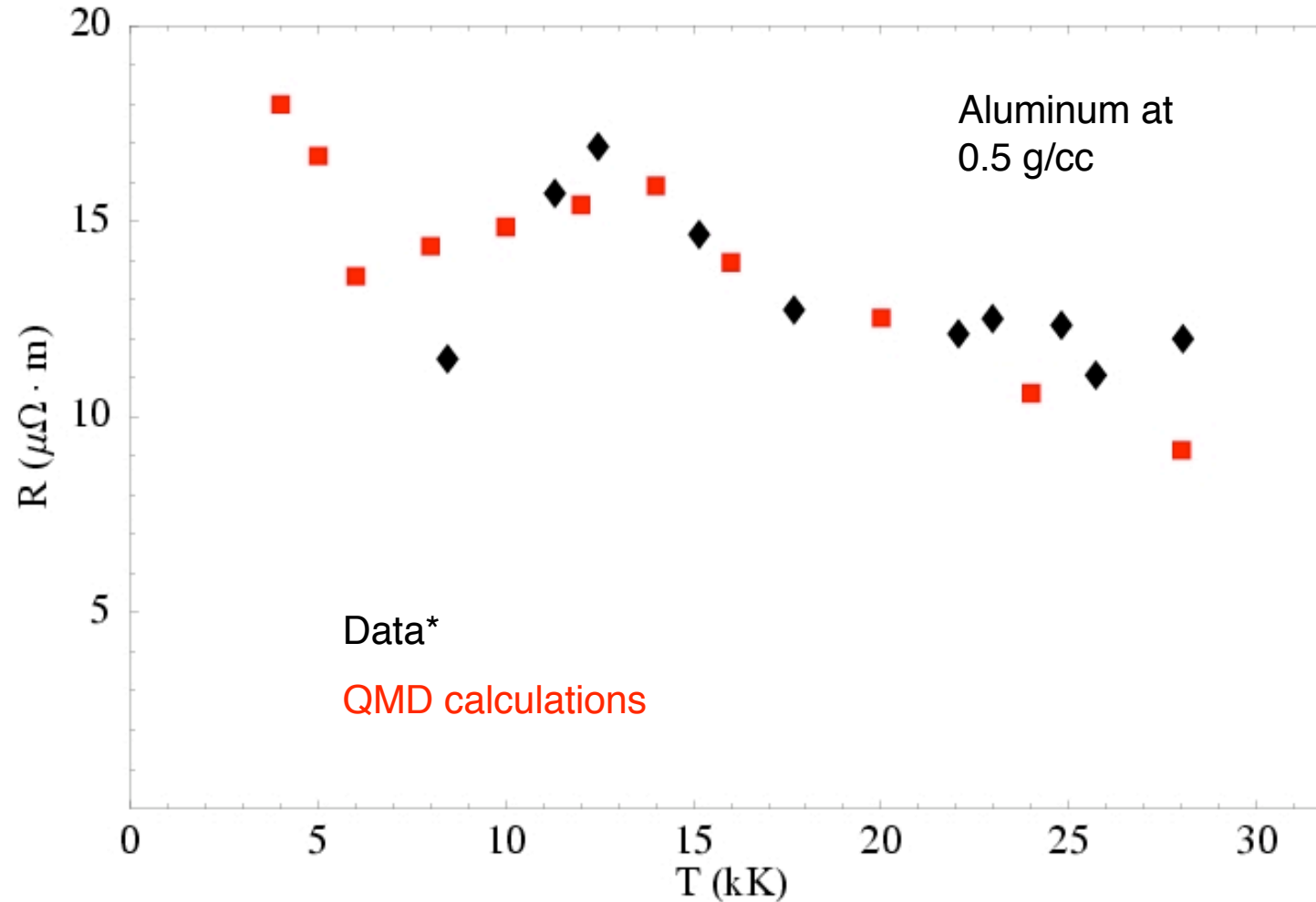
DeSilva and Katsouros data in black or grey, MD-KG results in red

★ 30000 K, ■ 20000 K, ▲ 10000 K, ◆ 6000 K

[Desjarlais, Kress, and Collins, PRE **66**, 025401(R) (2002)]



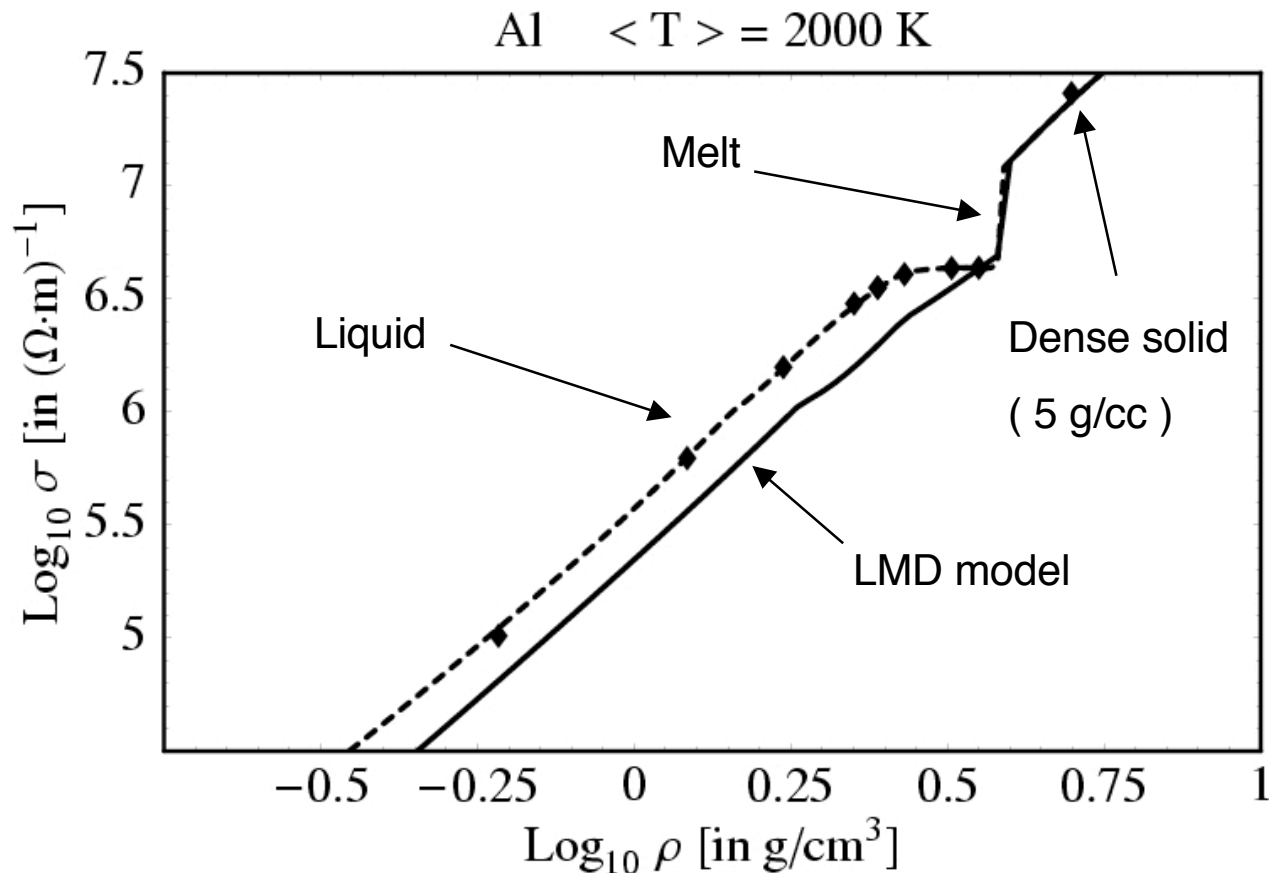
We find good agreement with isochore resistivity data from tamped exploding foil experiments\*



\*Data from Korobenko, Rakhel, Savvatimski, and Fortov, PRB 71 014208 (2005)



The calculated liquid aluminum conductivities are higher than the *improved* Lee-More (LMD) model predictions



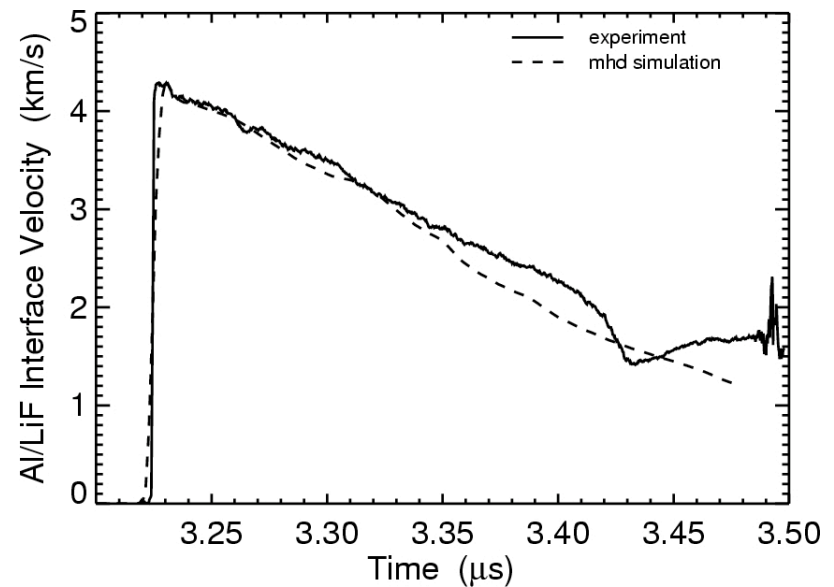
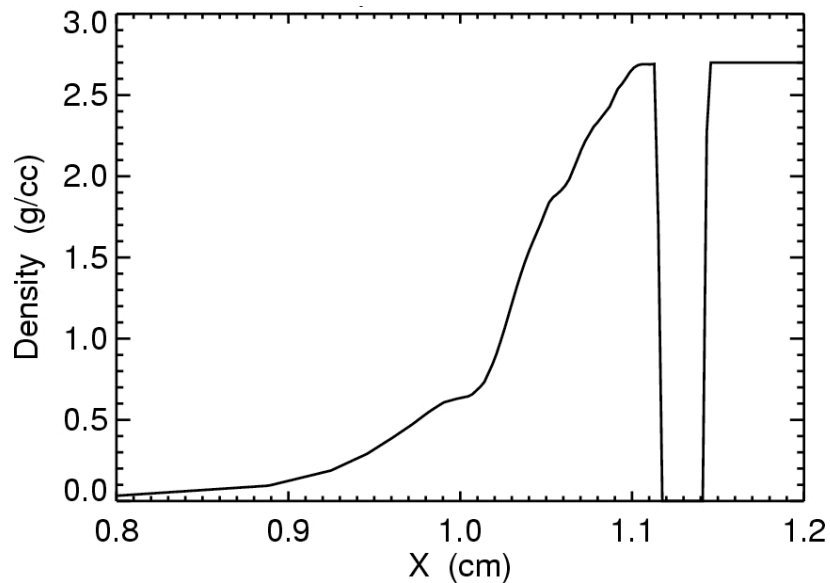
The dashed line shows the 2000 K isotherm from our QMD-tuned wide-range aluminum model



## Flyer plate simulations with the QMD based conductivities give very good agreement with experiment

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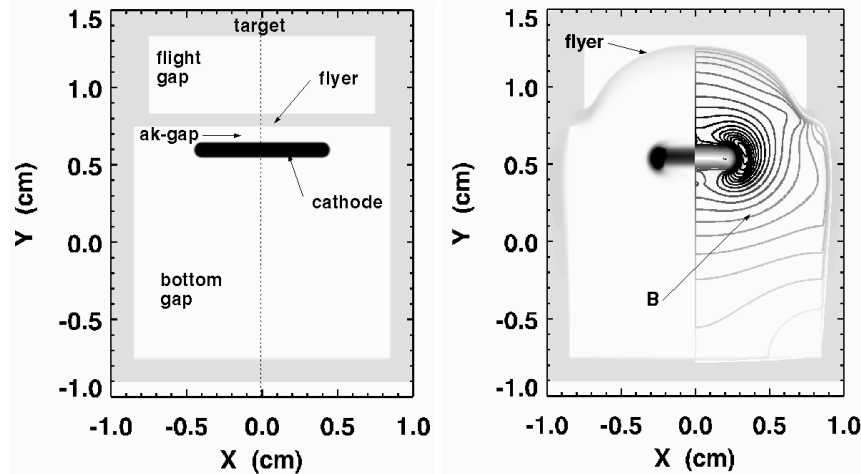
Simulations by Ray Lemke with Sandia's 3D Rad-MHD ALEGRA code



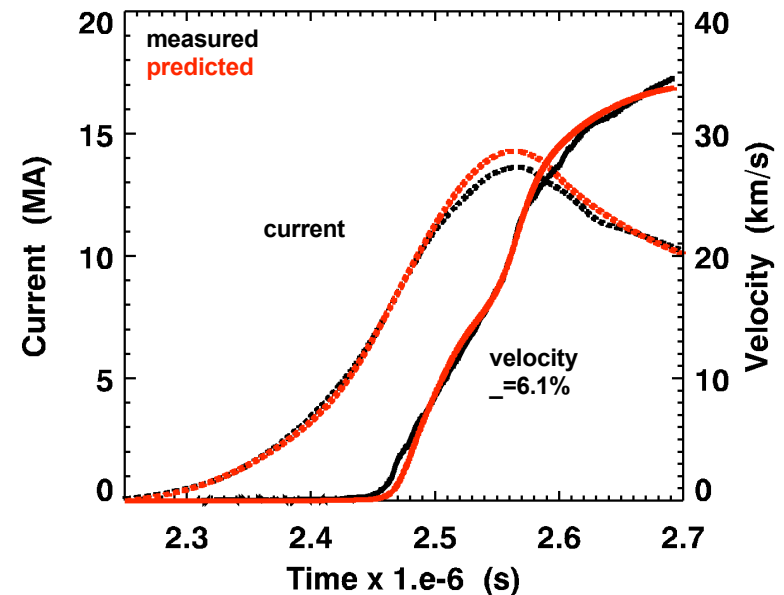
Conductivities based on the QMD calculations have given us a new predictive capability.



## We have used our simulation capability with the new aluminum model to optimize flyer performance on Z



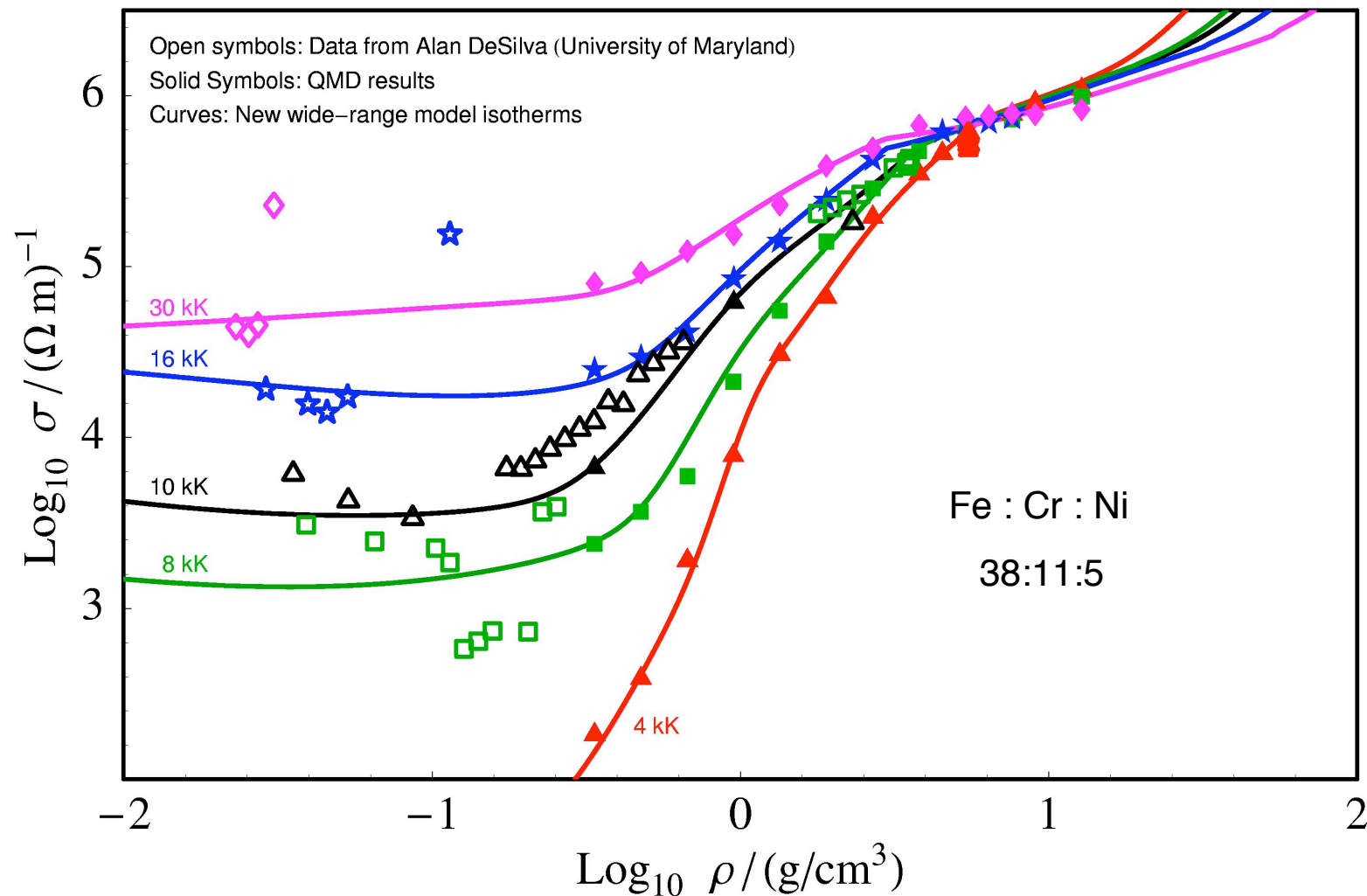
Measured / predicted current & flyer velocity (850  $\mu$ m Al)



The simulations were performed by Ray Lemke using Sandia's ALEGRA code in 2-D

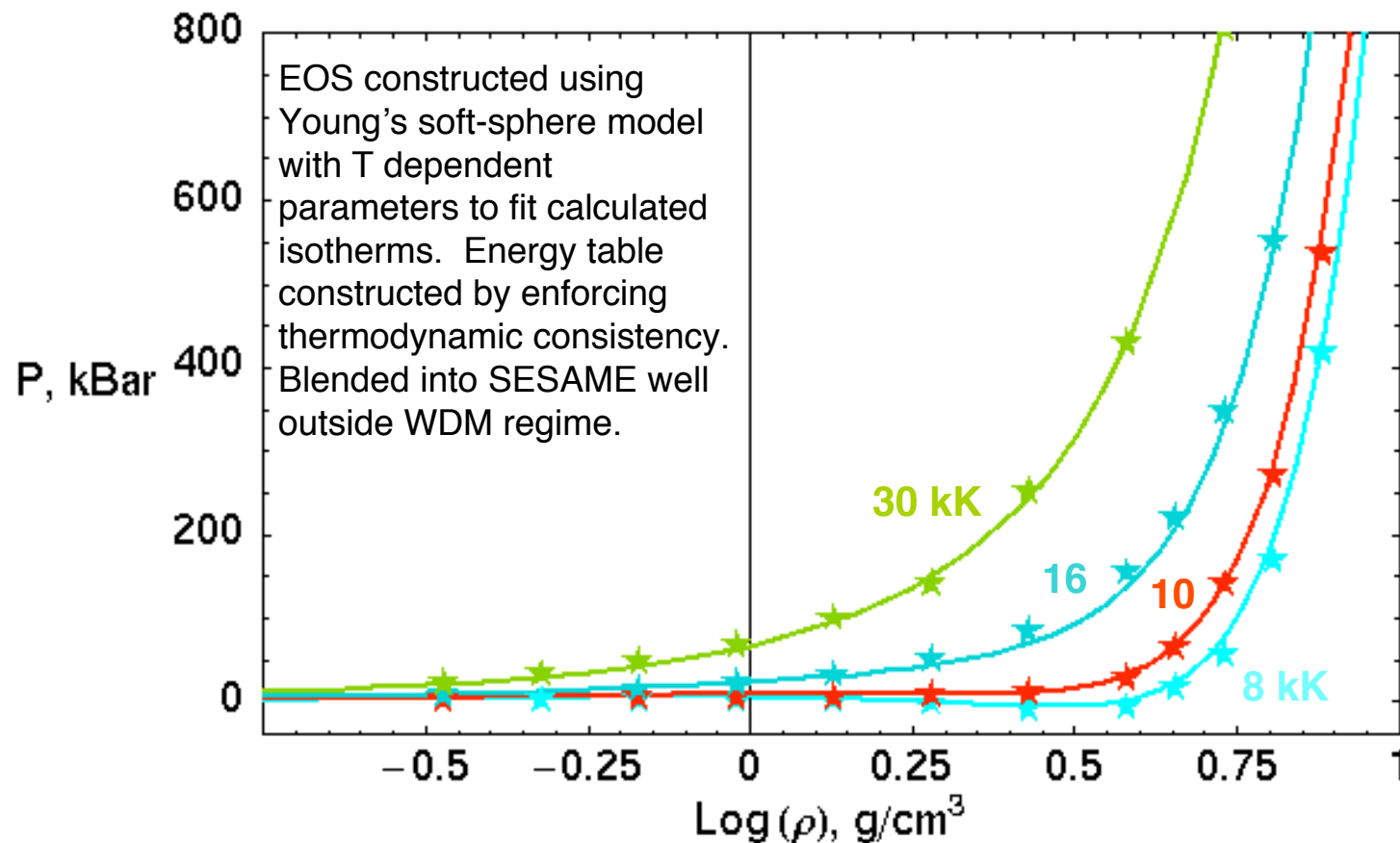


## We have applied the QMD/KG approach to develop a high fidelity conductivity model for stainless steel





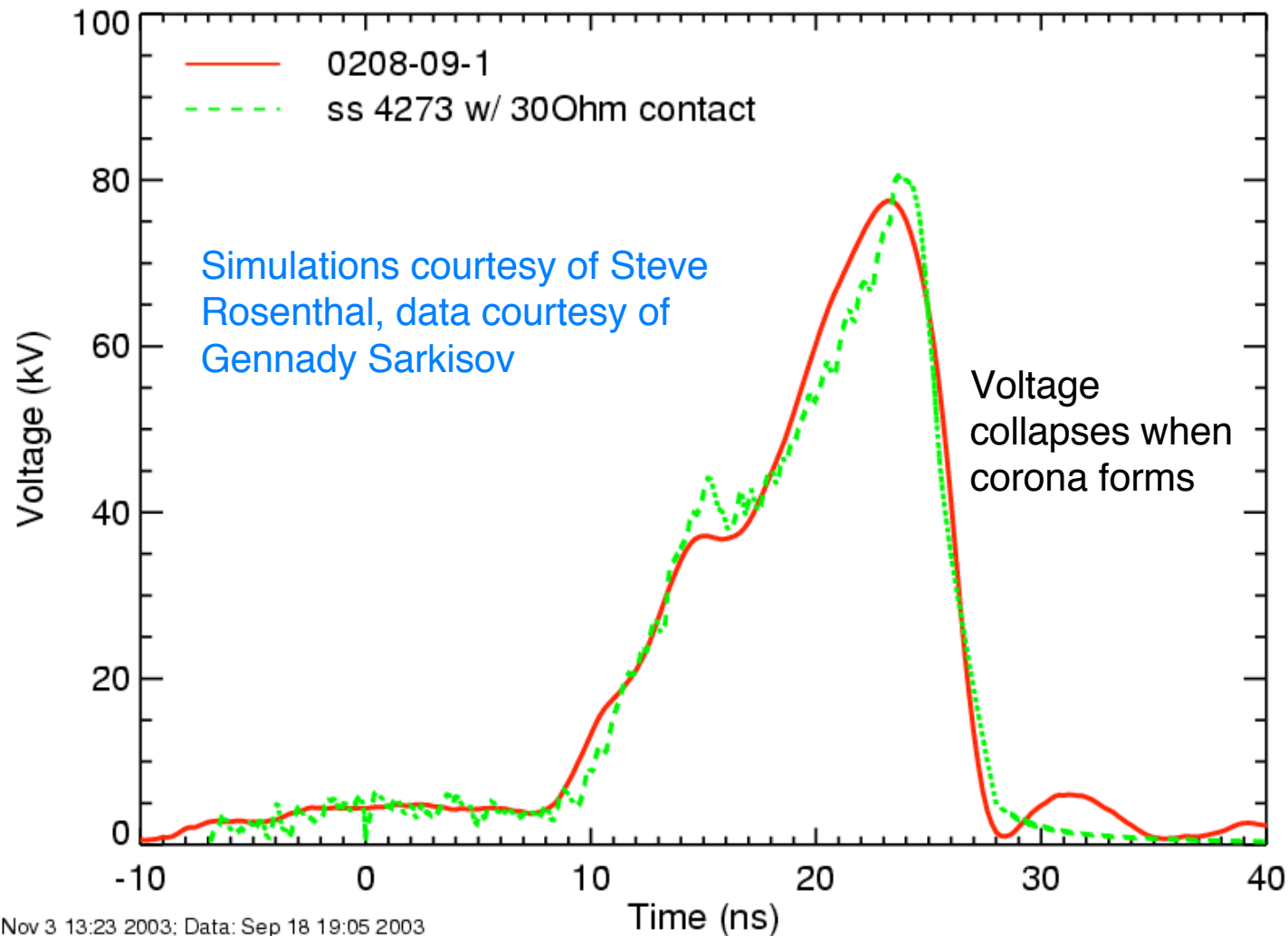
## We have used QMD to generate an Equation of State for Stainless Steel, consistent with the conductivity model



**We find the stainless steel liquid-vapor critical point to be at 10,000 K and 2.0 g/cc**



## Simulations of exploding stainless steel wires with the new models are in very good agreement with experiment



Plot: Nov 3 13:23 2003; Data: Sep 18 19:05 2003  
File: /remote/serosen/home/Alegra/wire/Gena/stainless/waveforms.pff



# Conductivity in water -- electronic conduction and proton conduction increase the complexity

## Electronic contribution

- Kubo-Greenwood theory
- 20-30 snapshots from a long QMD simulation
- Uniform and reproducible

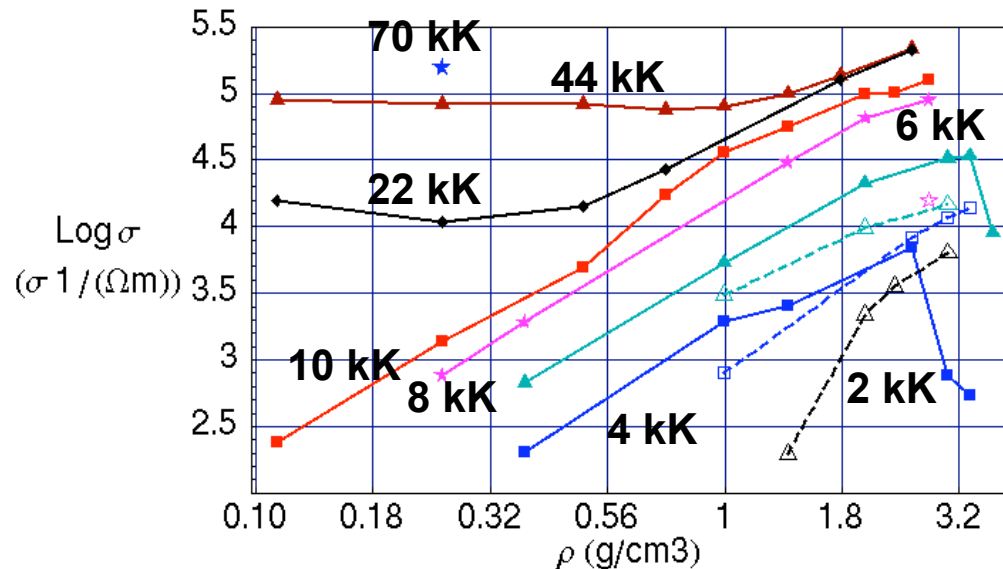
## Proton contribution

- Classical Kubo expression

$$\sigma = \frac{ne^2}{m} \int_0^\infty \frac{\langle v(\tau)v(0) \rangle}{\langle v(0)v(0) \rangle} d\tau$$

- 10-20 ps QMD simulations.
- Long enough simulations to obtain converged correlation functions

## Electrical conductivity of HEDP water



- Full lines: electronic conductivity
  - Suppressed in the superionic phase
  - Dominates above 6000 K
- Dashed lines: proton conduction
  - Dominates below 4000 K
- Quantitative conductivity over a large region of phase-space



**We are using QMD/DFT calculations to produce accurate compression and release isentropes**

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$$dS = \frac{\partial S}{\partial T} dT + \frac{\partial S}{\partial V} dV$$

$$\frac{\partial S}{\partial T} = \frac{1}{T} \frac{\partial E}{\partial T}$$

$$\frac{\partial S}{\partial V} = \frac{\partial P}{\partial T}$$

$$\frac{1}{T} dT = - \frac{\partial P / \partial T}{\partial E / \partial T} dV = - \frac{\partial P}{\partial E} \bigg|_V dV$$

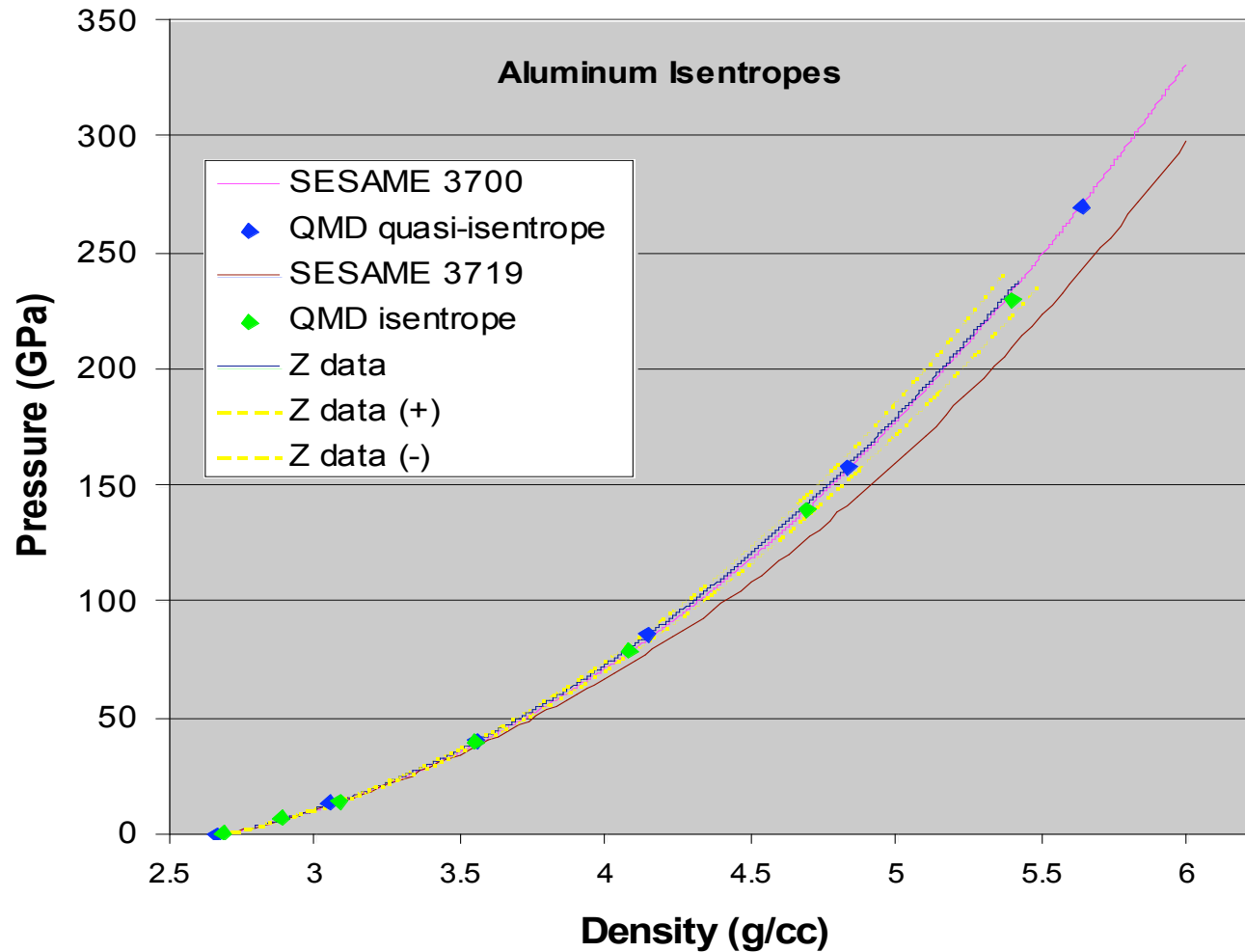
Piecewise integration gives

$$\frac{T_{i+1}}{T_i} = \exp \left[ - \int_{V_i}^{V_{i+1}} \frac{\partial P}{\partial E} \bigg|_V dV \right]$$

**We can make direct comparisons between the QMD isentropes and data from Isentropic Compression Experiments (ICE) on Sandia's Z machine**



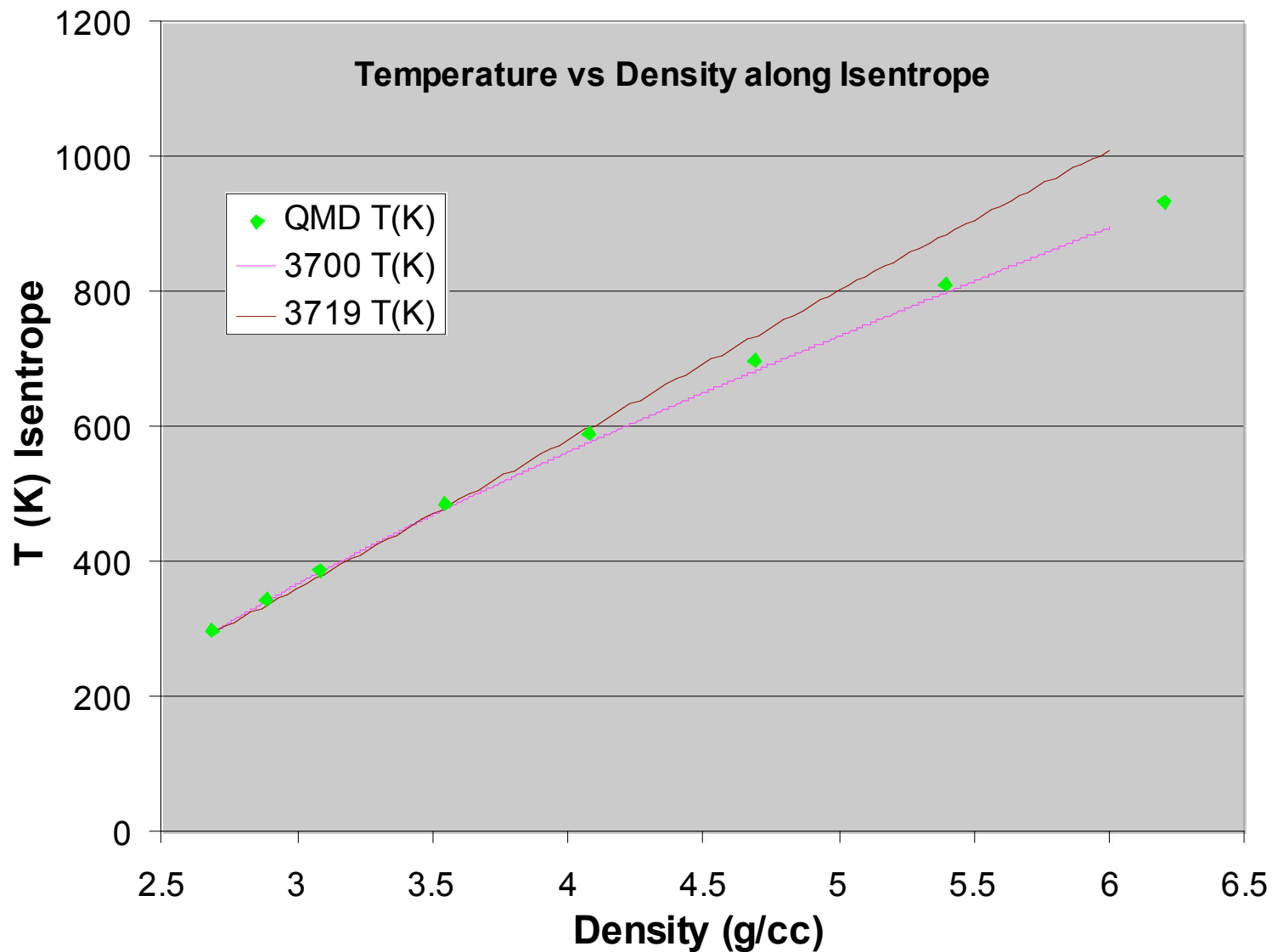
## Our calculated principal isentrope for aluminum is in excellent agreement with data from Sandia's Isentropic Compression Experiments (ICE) on Z



Data from **ICE**  
experiments on  
Z courtesy of  
Jean-Paul Davis

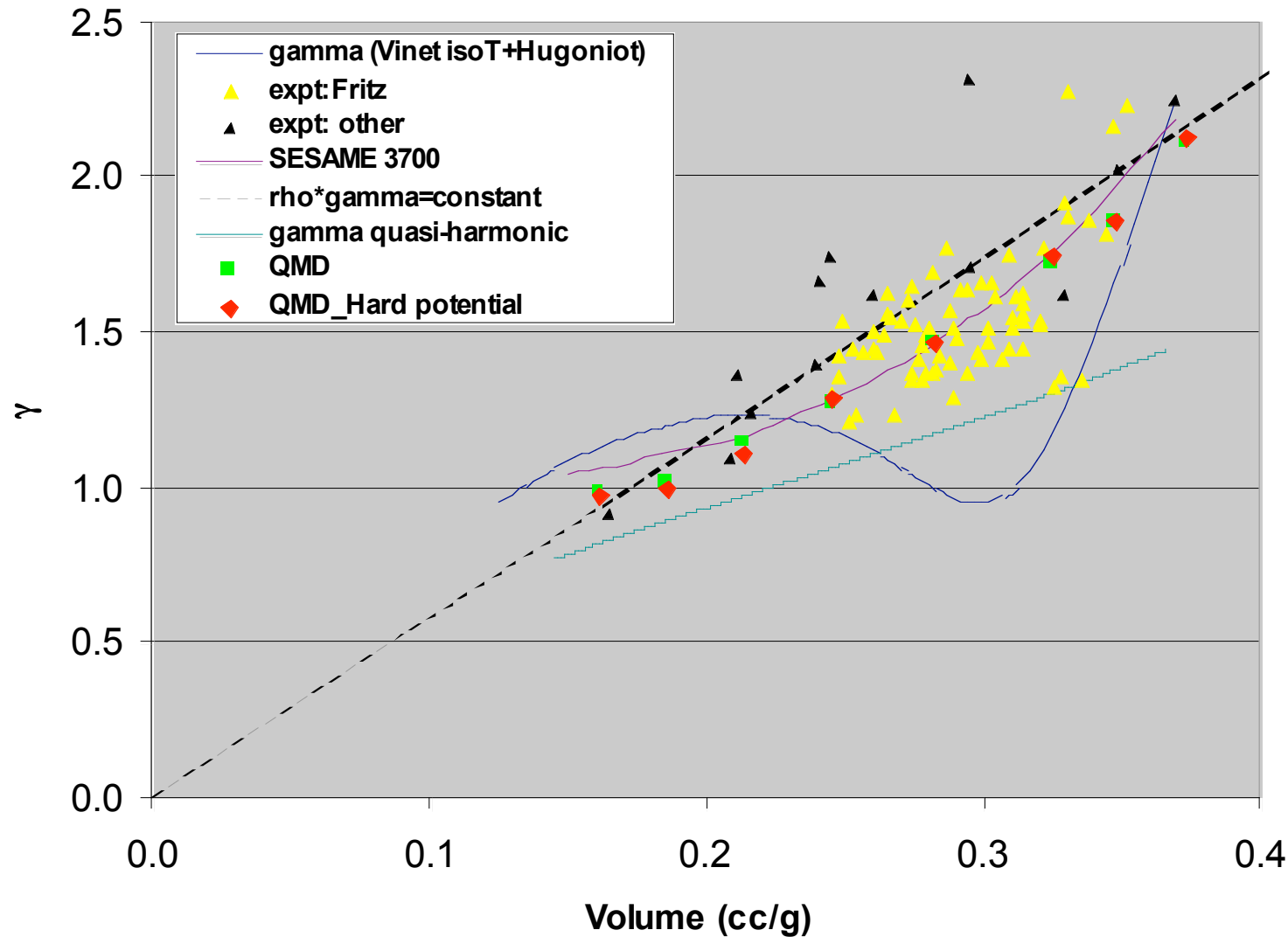


**The temperature versus density along the isentrope is also in excellent agreement with SESAME 3700 (Kerley)**





Our calculated Gruneisen  $\gamma$  for solid aluminum is in very good agreement with data



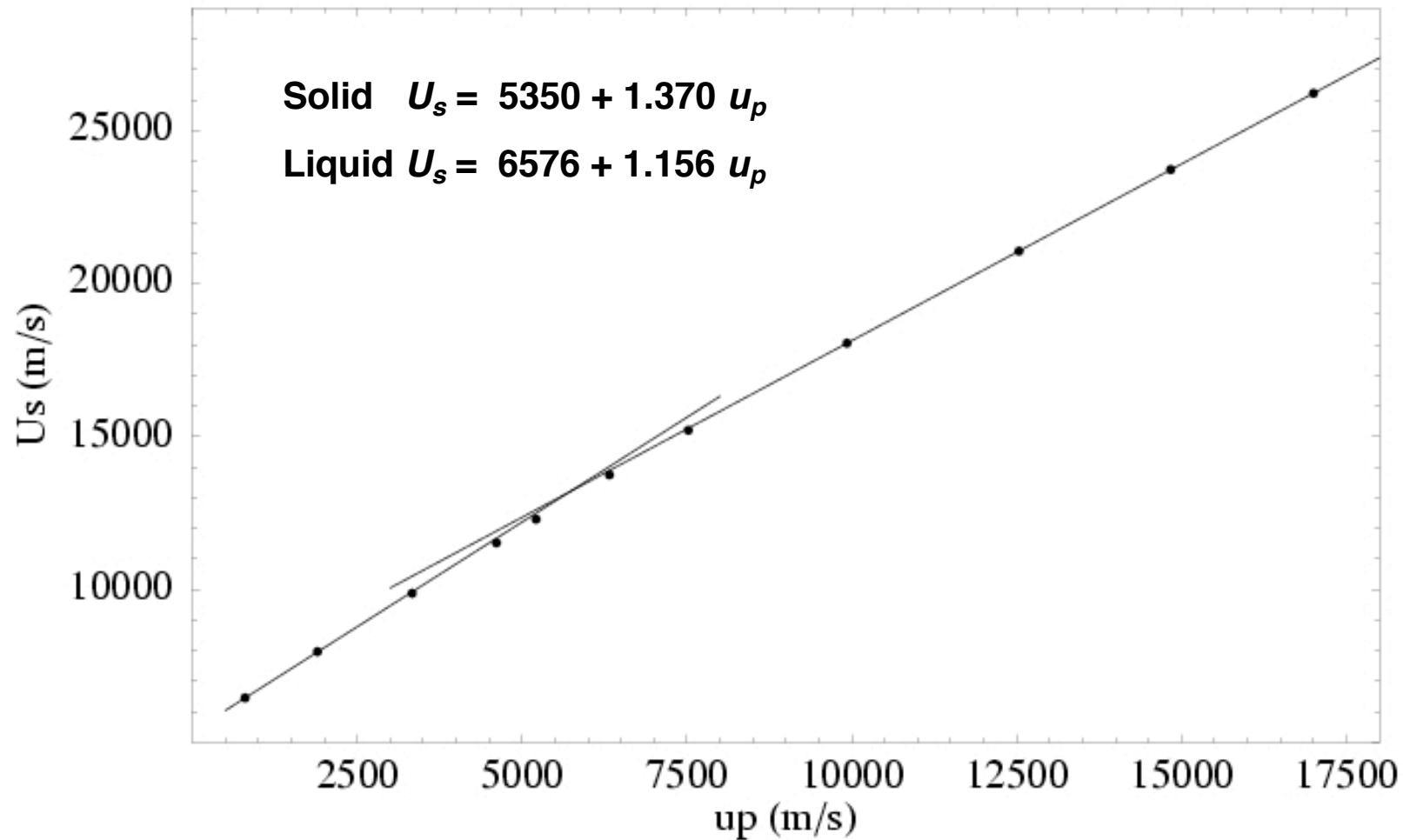
$$\gamma \equiv V \frac{\partial P}{\partial E}$$

Data and model  
compilation  
courtesy of  
Dennis Hayes



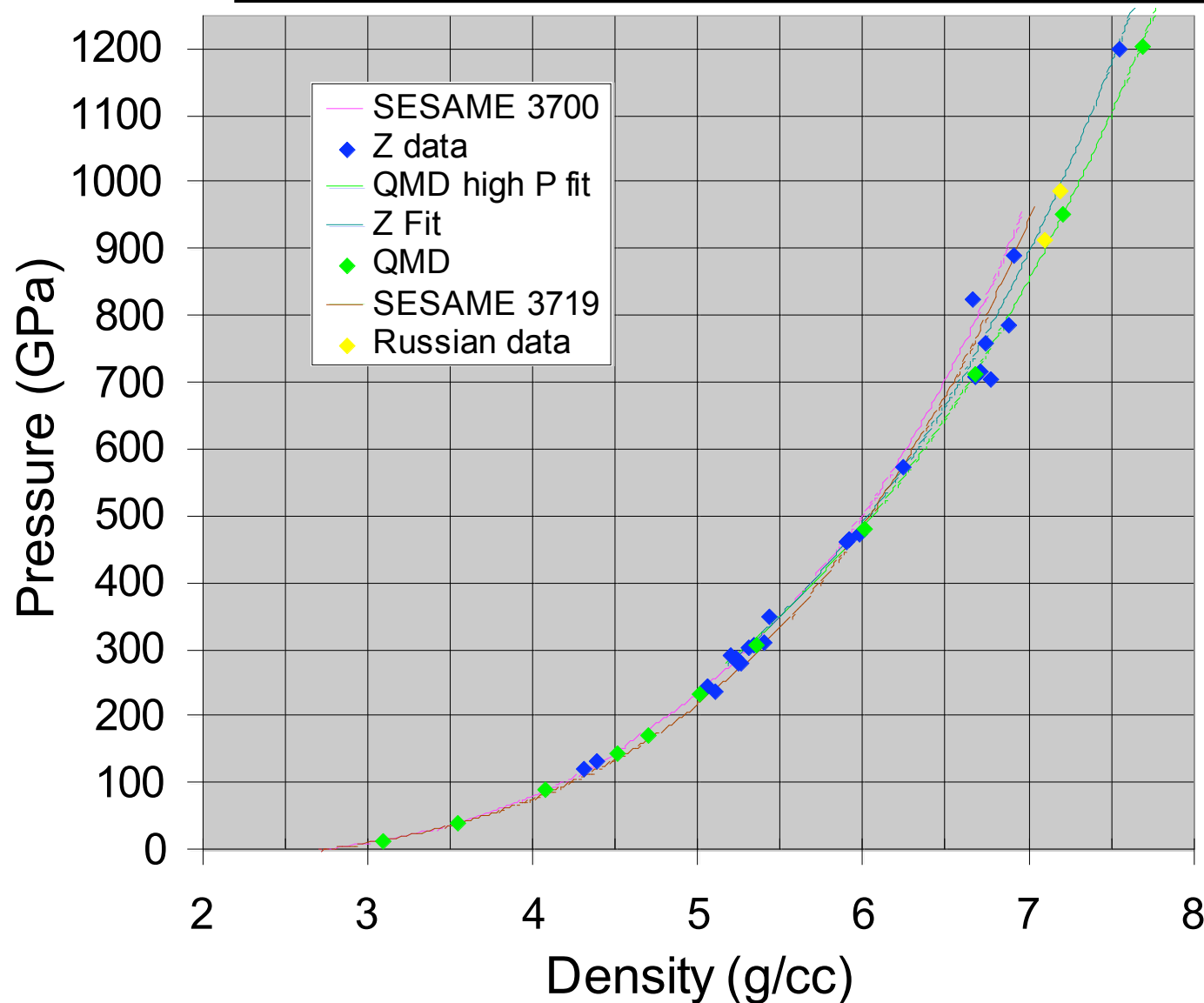
**We have calculated the aluminum Hugoniot from solid density up to 12 Mbar**

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## Our aluminum Hugoniot is in very good agreement with data

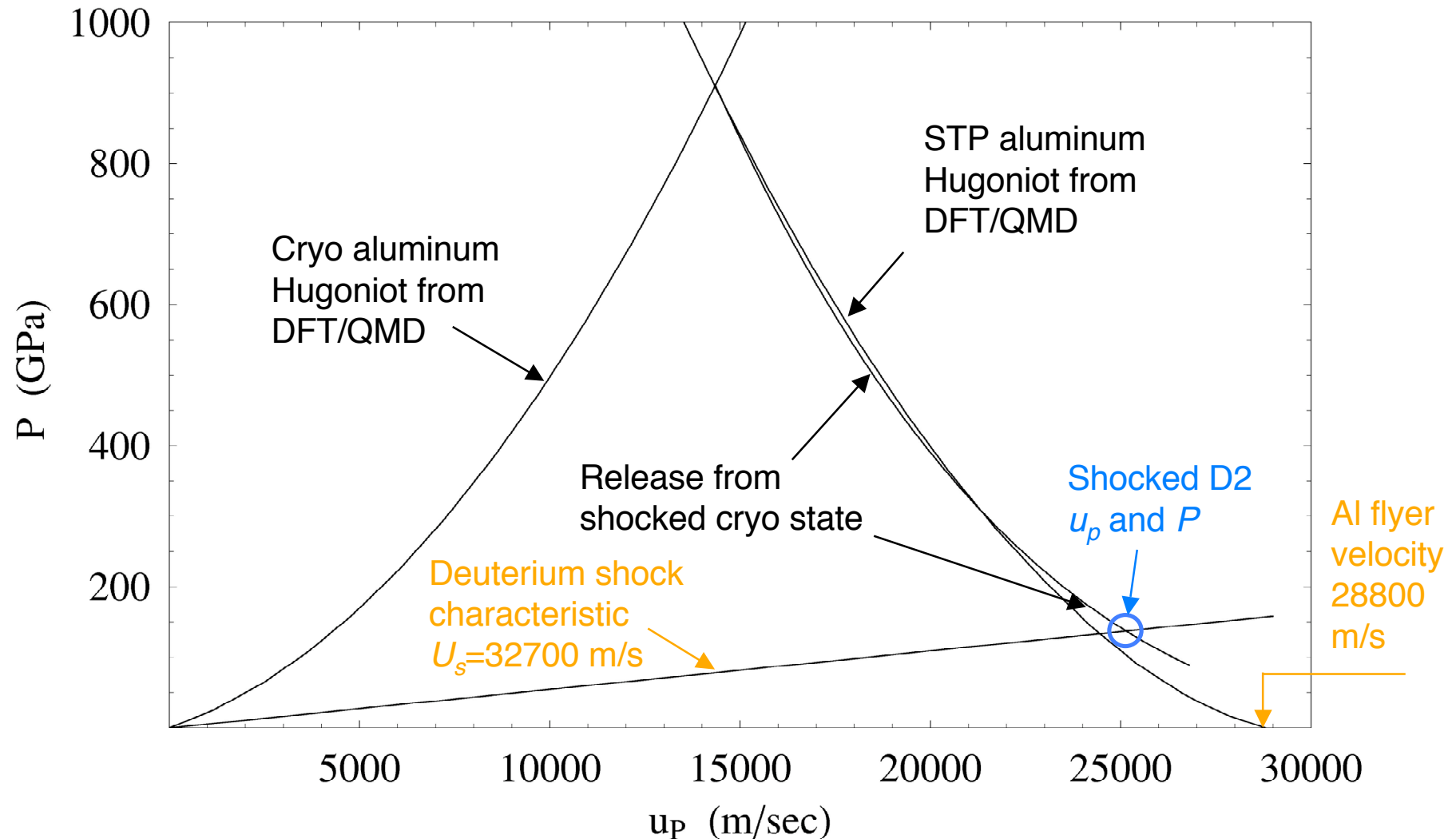


3700 appears to give a good isentrope, but the Hugoniot is too stiff

Al Hugoniot data  
courtesy of  
Marcus Knudson

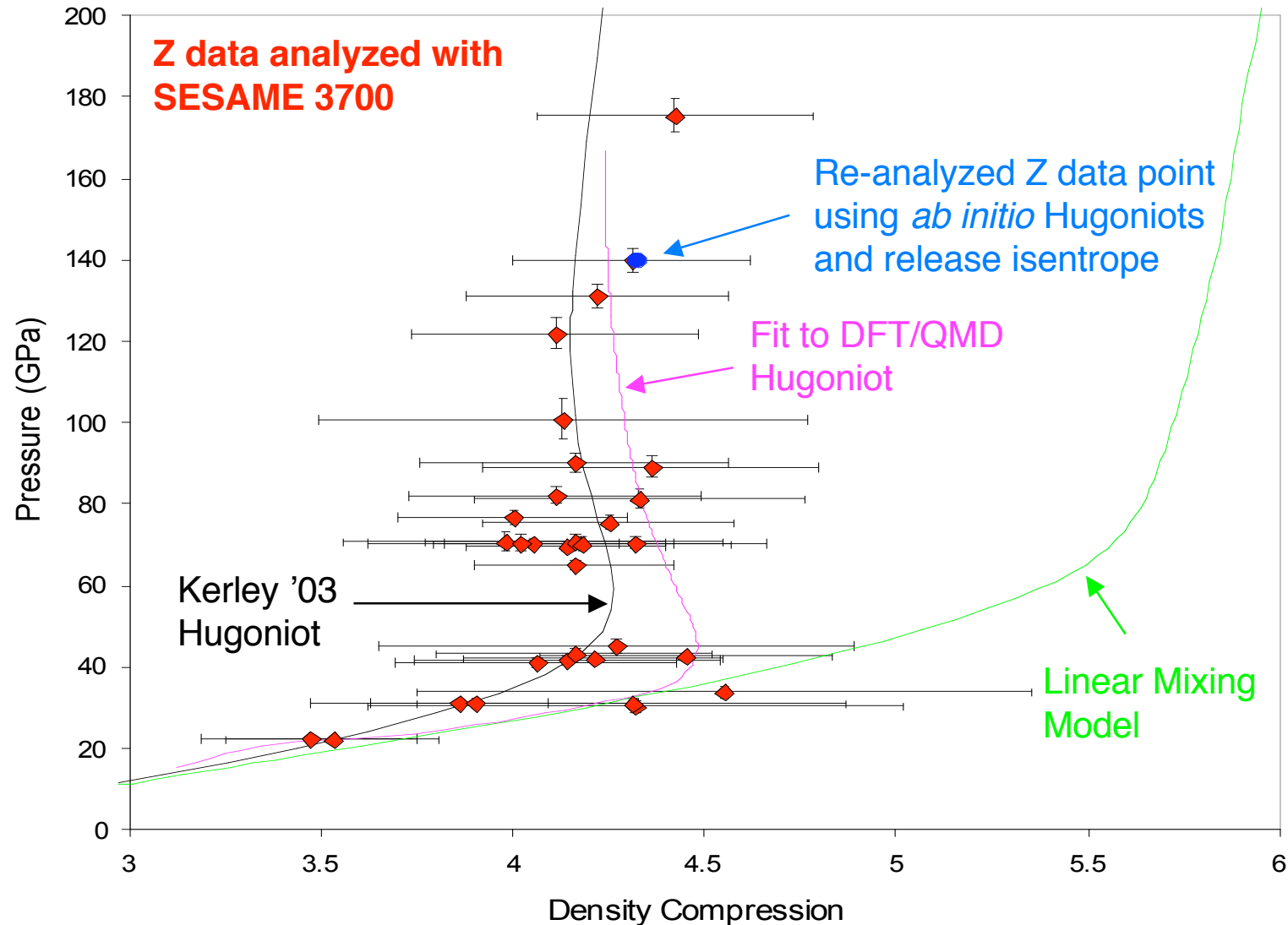


## We have all the pieces to do *ab initio* impedance matching calculations



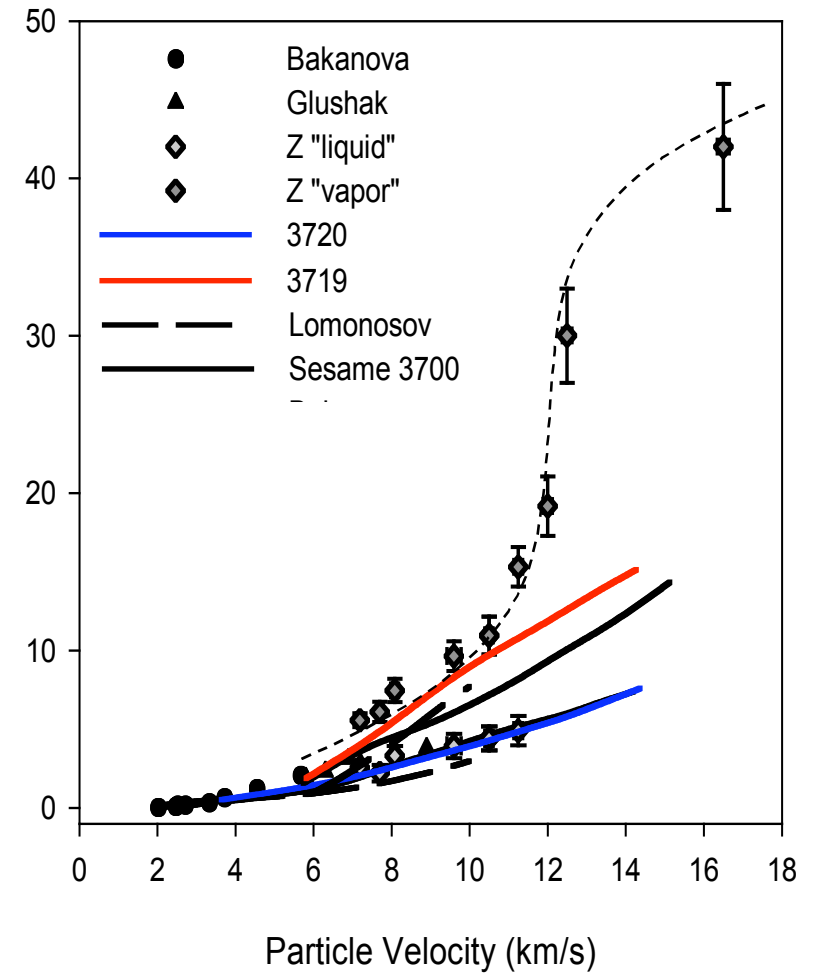
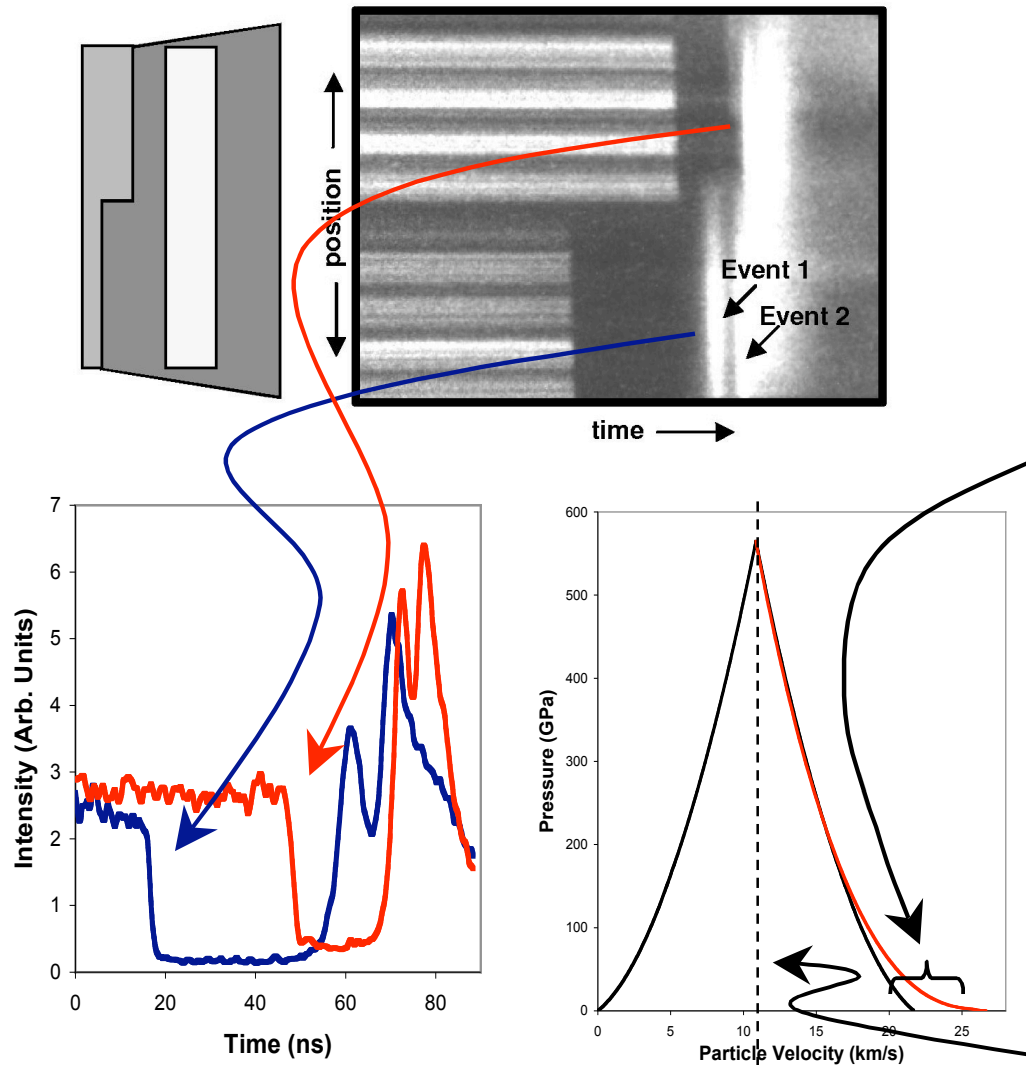


**Our re-calculated impedance match point is very close to that obtained with SESAME 3700**





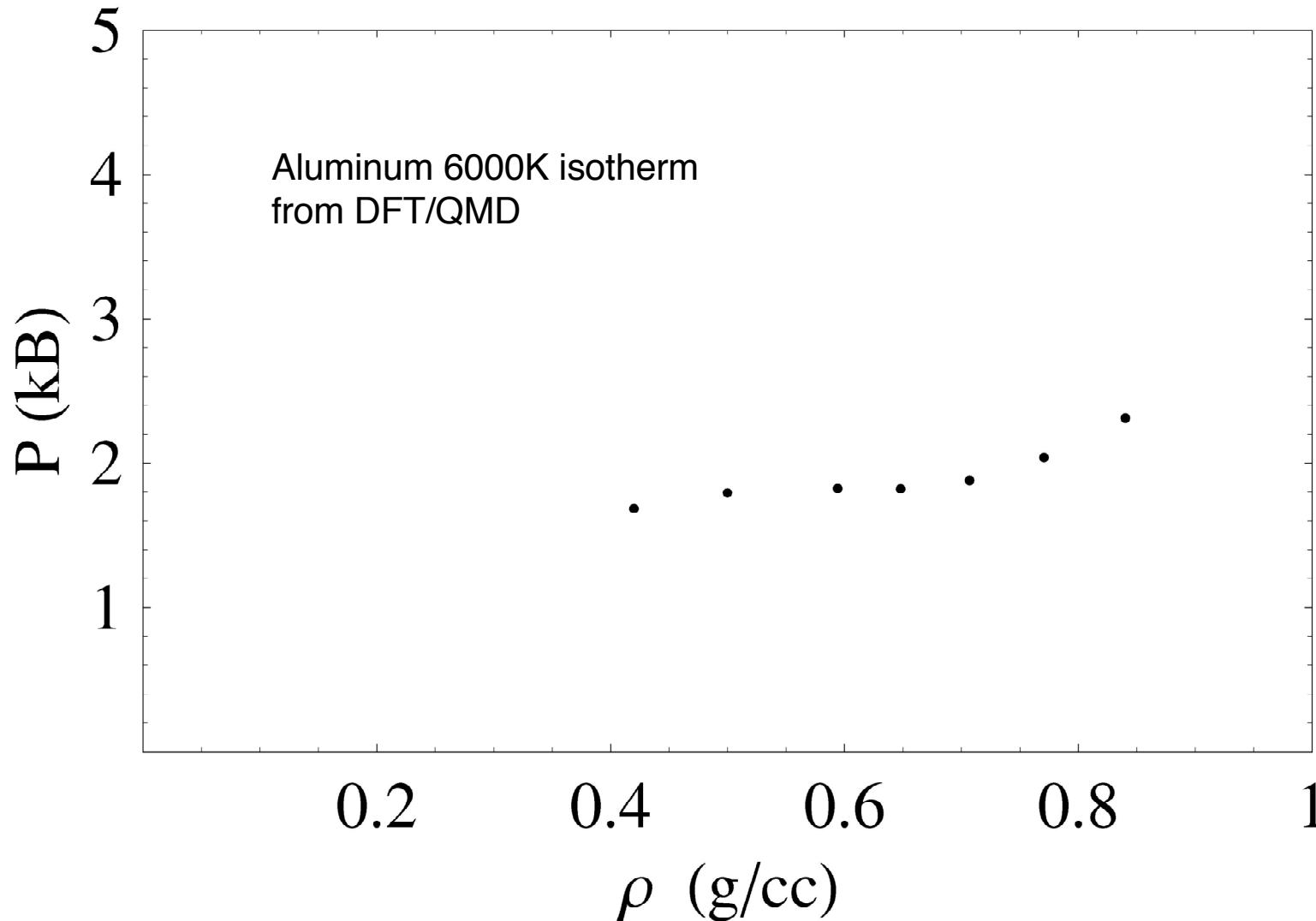
## Full release measurements are being performed on Z





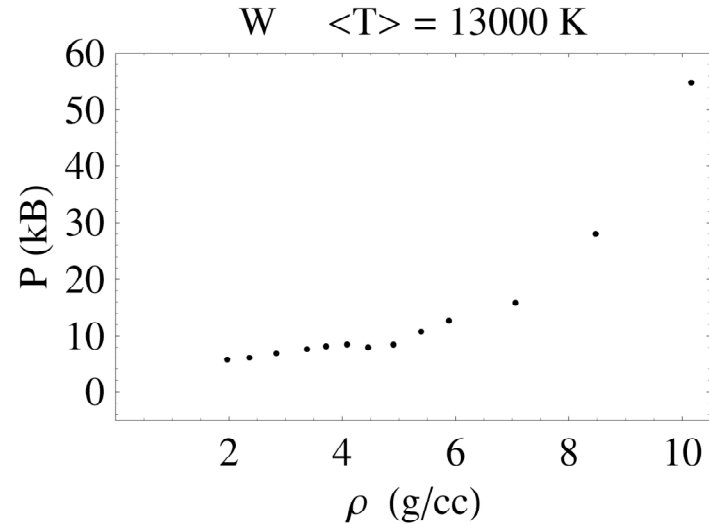
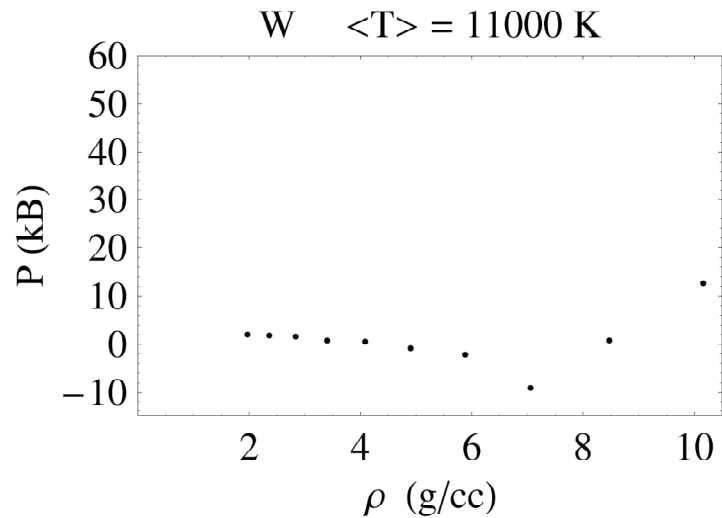
**Our calculated liquid-vapor critical point for aluminum appears to be very close to 6000 K, and at 0.6 g/cc**

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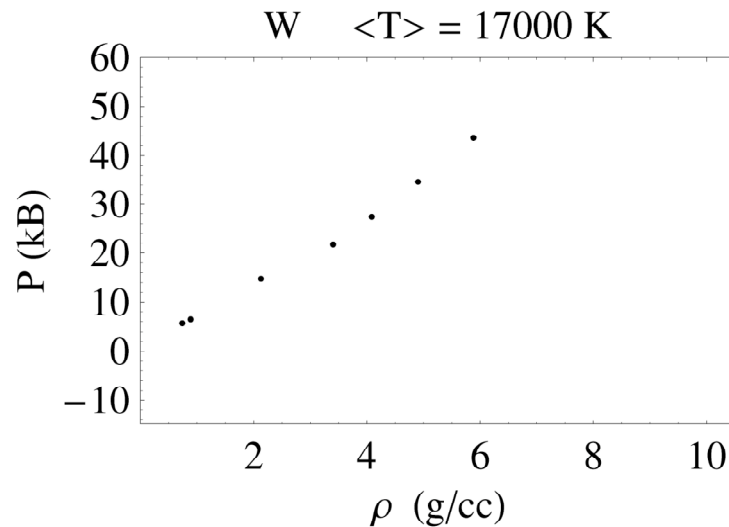
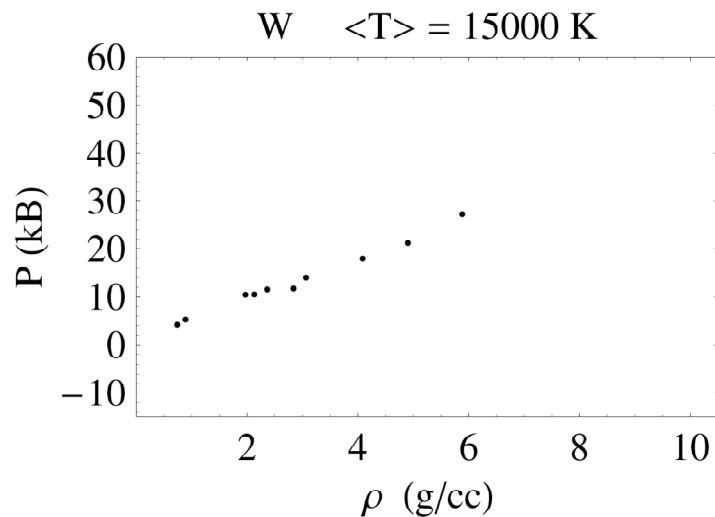


## Isotherms for W have been calculated for EOS development in collaboration with Richard More



Liquid-vapor  
critical point

$\sim 13000$  K  
 $\sim 4.5$  g/cc





## Recent and active research areas

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- QMD based conductivity models for Al, W, Be, and SS
- Reflectivity of shocked xenon
- Principal Hugoniot and reshock properties of deuterium
- Principal and release isentropes of Al
- Principal Hugoniot of Al and W
- Liquid-vapor critical points (Al, W, SS)
- Electrical and thermodynamic properties of water at high energy densities
- Equations of state for SS, Al, and W
- Research on advanced electronic structure methods for HEDP (finite temperature Exact Exchange, finite temperature GW)