Beam Stability and Control in Solenoid Transport Channels*

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Outline:

Part I: Stability (Mostly Review):

Transverse Beam Stability in Periodic Solenoid Transport

- Envelope Modes
- Higher Order (Kinetic) Effects

Coworkers:

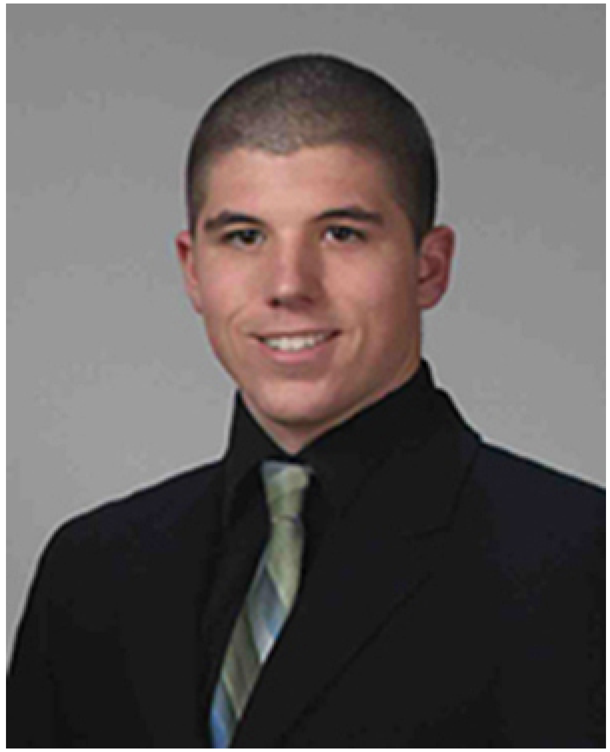
J.J. Barnard (LLNL)	B. Bukh (LBL)
S.R. Chawla (UCB)	S.H. Chilton (UCB)

Part II: Control (New)

Centroid Oscillations and Steering in Solenoidal Transport

Coworkers:

Theory:	E.P. Lee (LBL)	C.J. Wootton (UCB)
Modeling:	I. Haber (UMD)	W.M. Sharp (LLNL)
Experiment:	J.A. Coleman (UCB)	S.M Lidia (LBL)
	P.A. Seidl (LBL)	



Chris Wootton, a talented Nuclear Engineering student at UC Berkeley, was tragically killed just before graduation on May 3, 2008

Chris is missed by our group as both a friend and rapidly developing colleague. His contributions to this study in solenoid steering are duly noted.

“Good” transport of a single component, unbunched beam with intense space-charge and a reasonably smooth initial distribution requires:

Lowest Order:

1. Stable single-particle centroid controlled/steered to near-axis
   Cover in Part II

Next Order:

2. Stable rms envelope in moment models

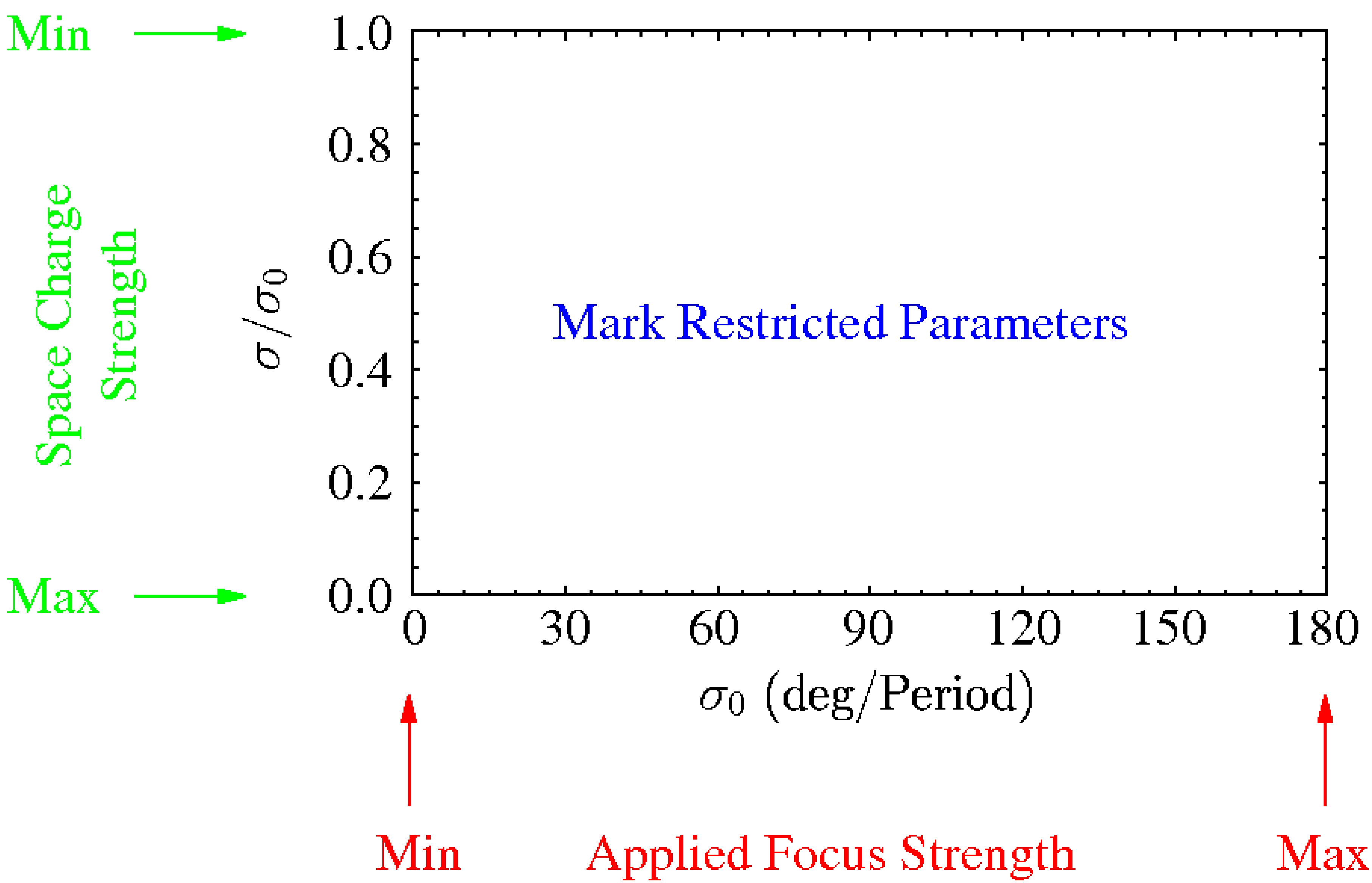
Higher Order:

3. “Stable” higher-order Vlasov description without
   large emittance growth and particle losses

Need to understand how these constraints restrict possible parameters to design intense beam machines in a solenoidal transport lattice

- Contrast 2 and 3 to quadrupole focusing

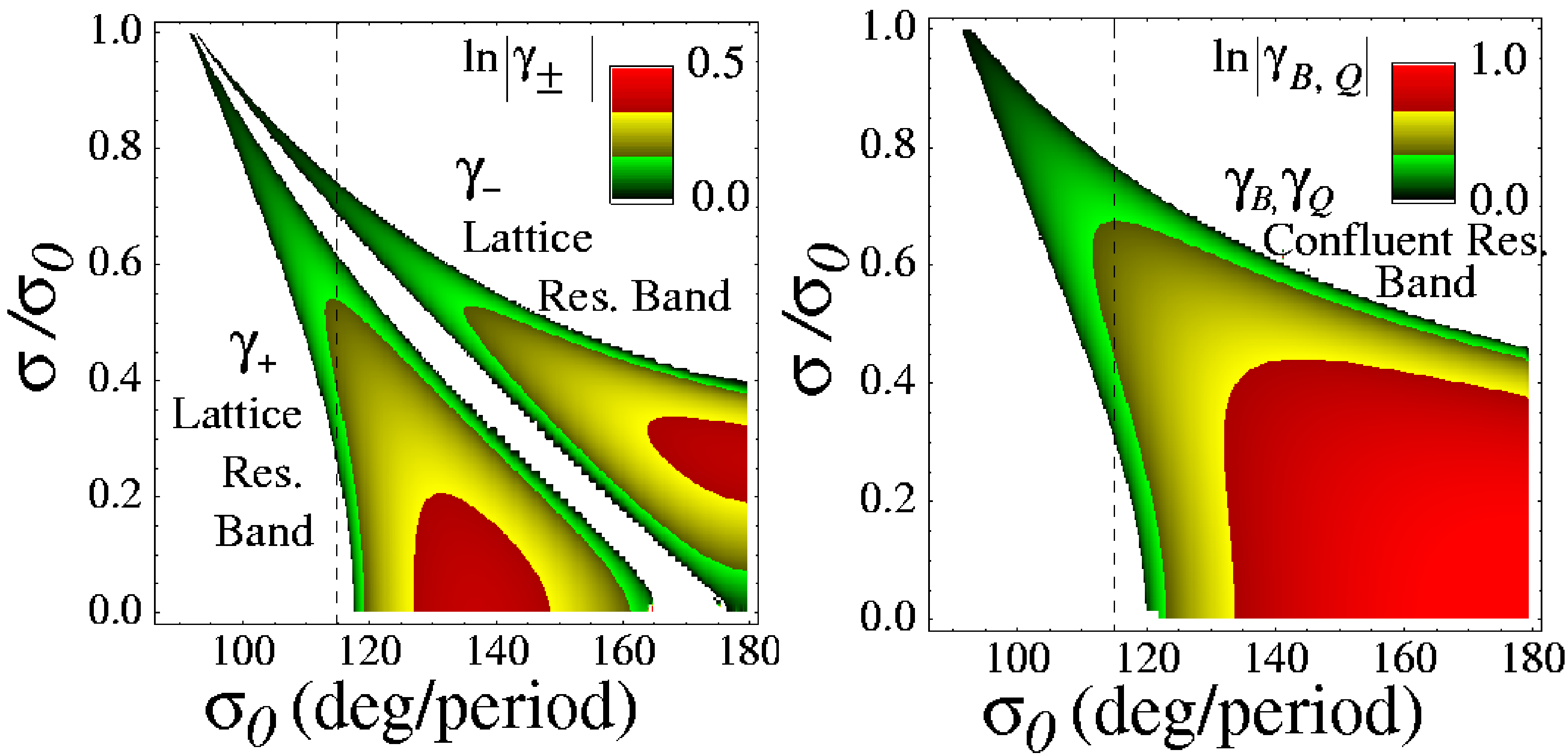
Examine points in operating parameters that can lead to problems ....



Instability bands of the KV envelope equation are well understood in periodic focusing channels and must be avoided in machine operation

Envelope Mode Instability Growth Rates

Solenoid (η = 0.25) Quadrupole FODO (η = 0.70)

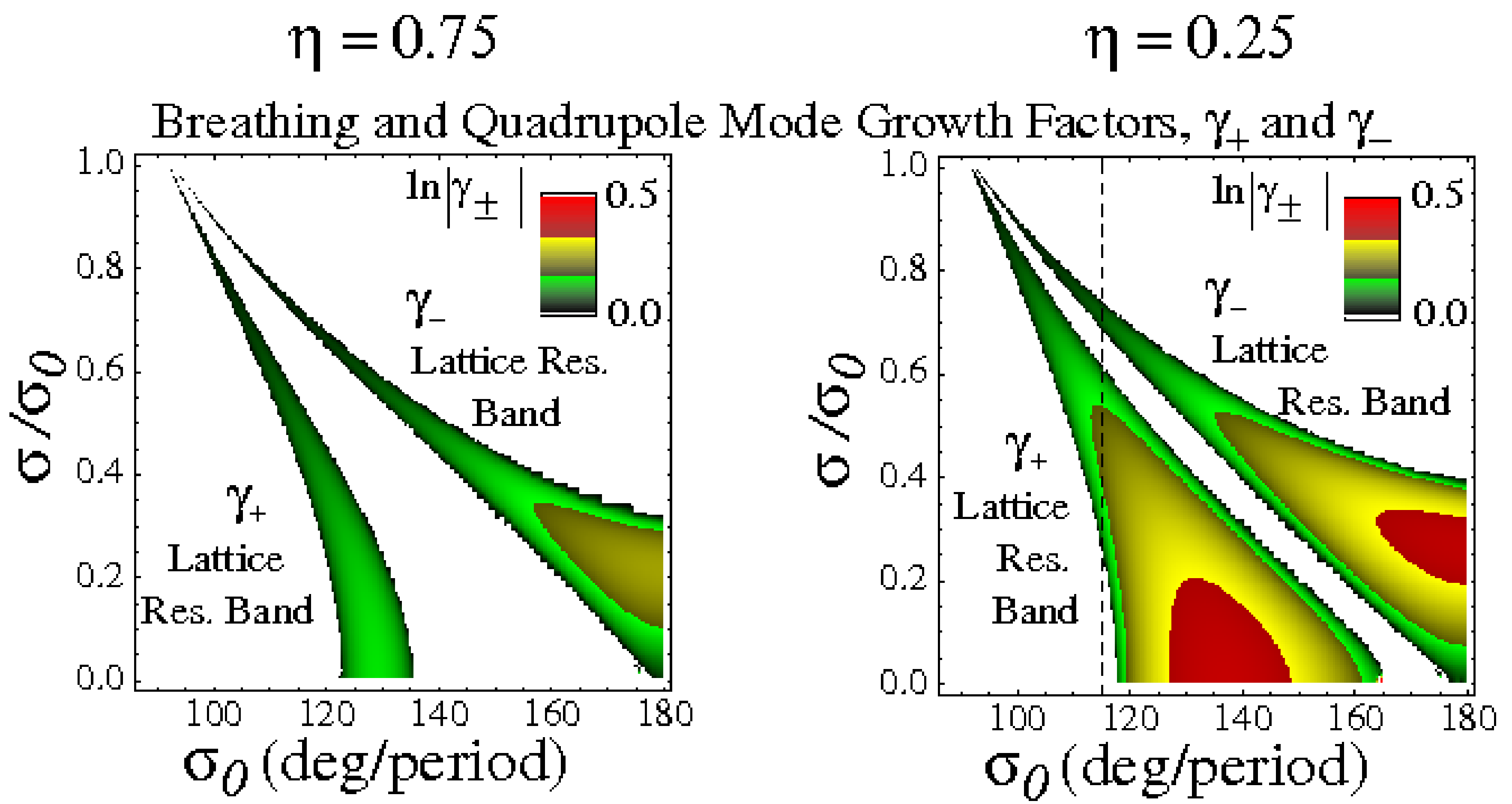


Lund and Bukh, PRSTAB 024801 (2004)

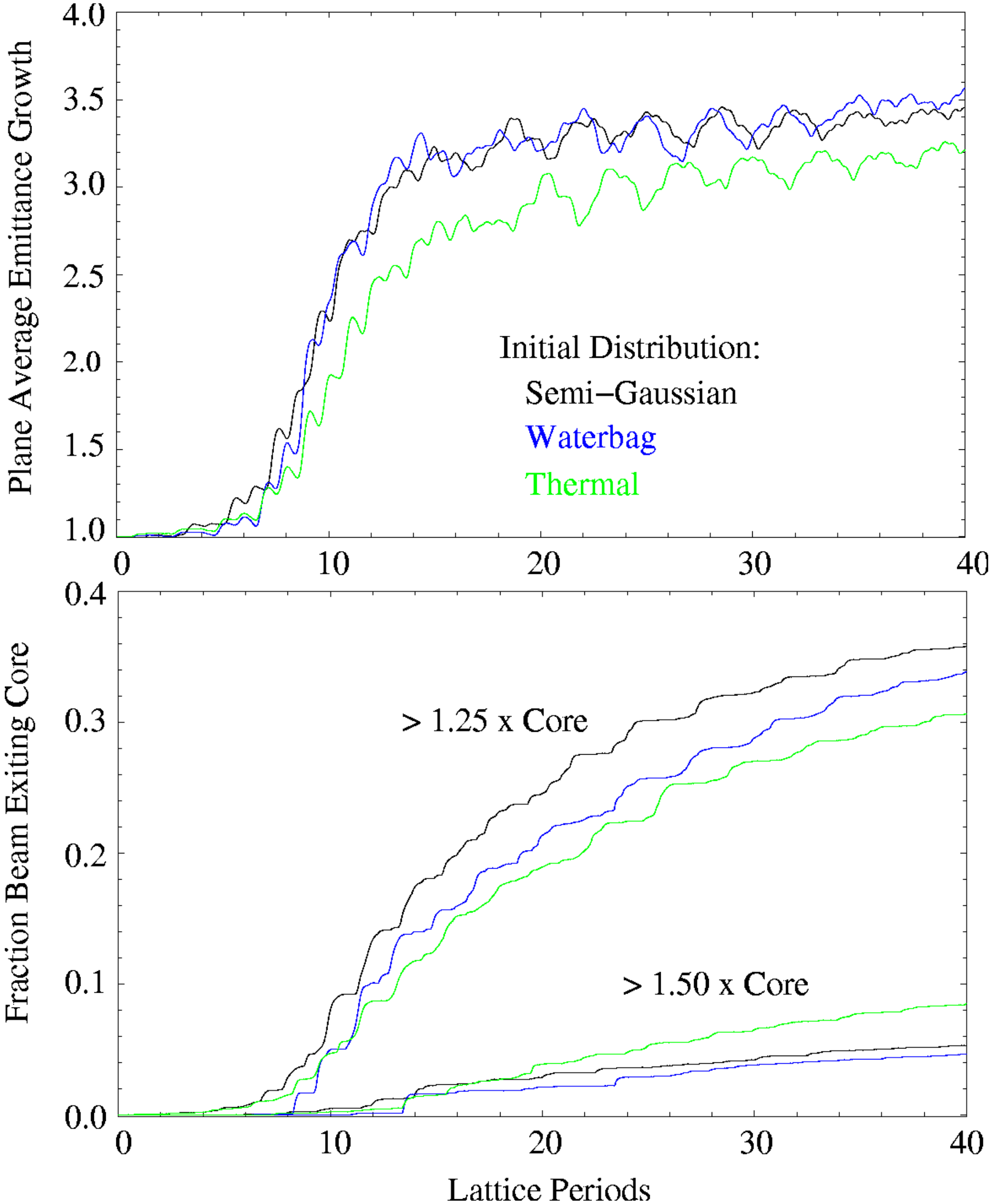
For solenoids, envelope instabilities strongly dependent on occupancy and become weaker as the solenoids fill the lattice

High occupancy solenoids used with space-charge strong

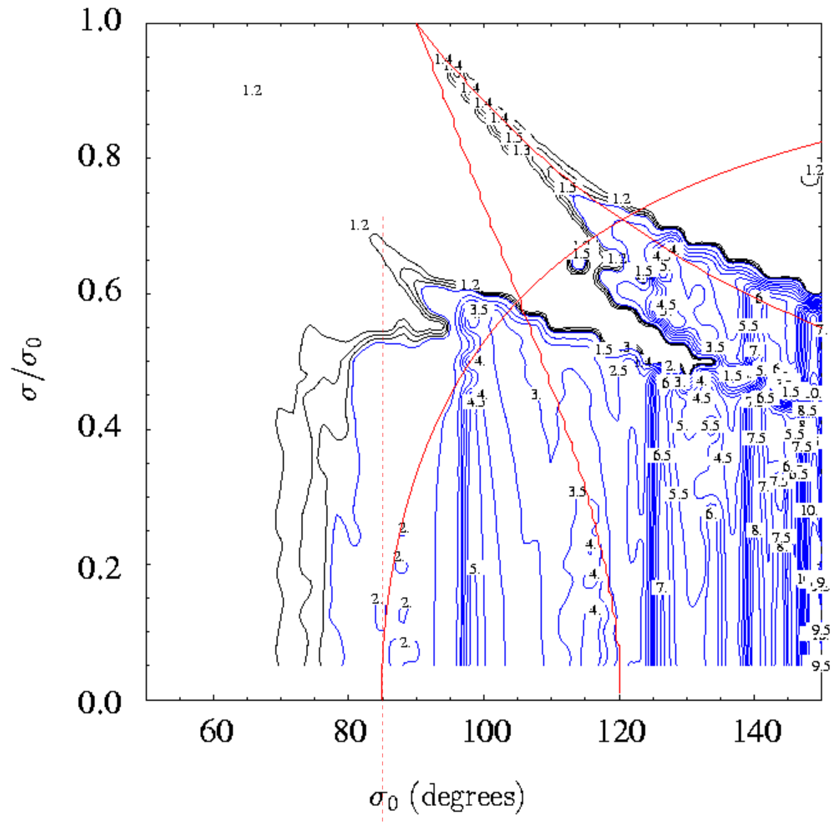
- Envelope modes weaker for solenoidal transport



Recent theory (simulations and reduced core-particle models) show that quadrupole transport is limited by strong halo processes



Parameter Restrictions



$$\sigma_0 \sim 85^\circ$$

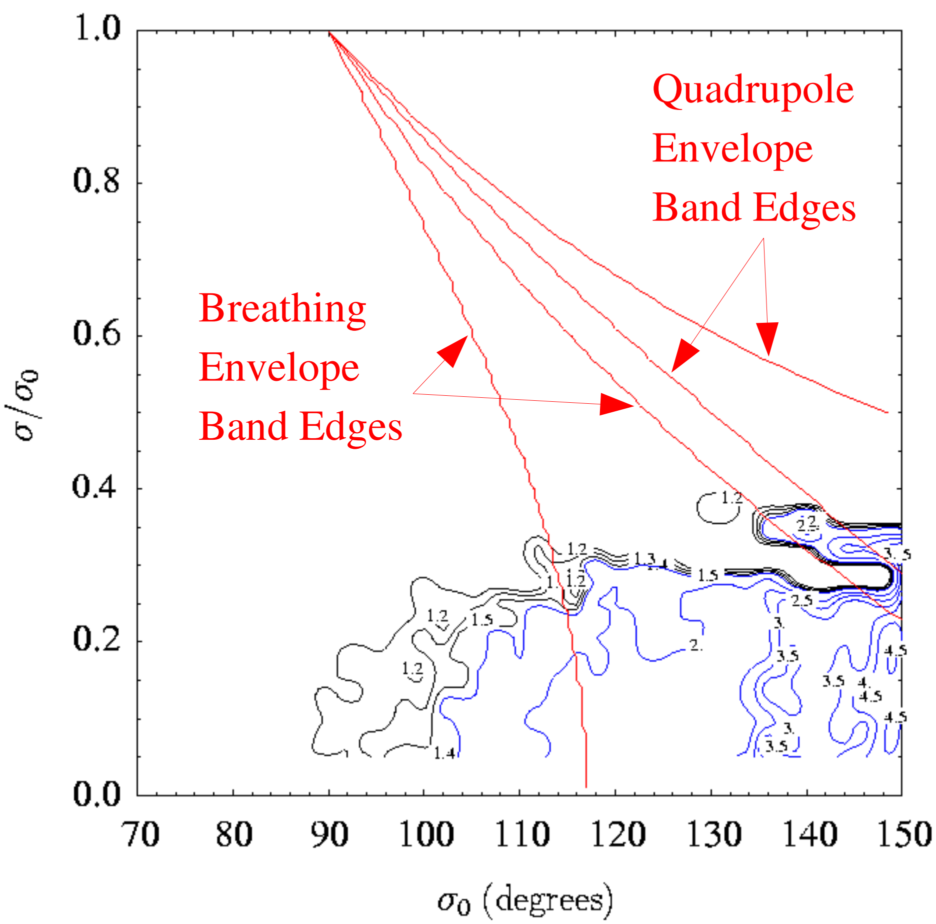
NIMA **561**, 203 (2006)

NIMA **577**, 173 (2007)

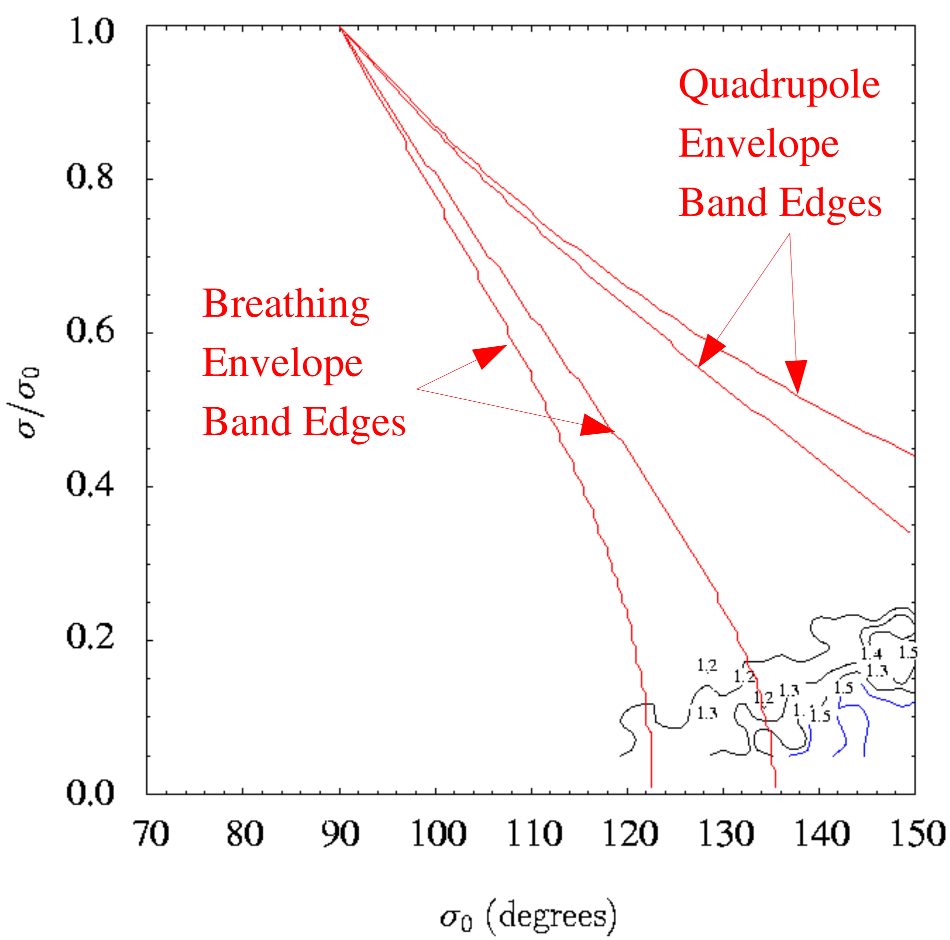
What about analogous processes for solenoidal transport?
Does an analog of the Tiefenback current limit exist?

Matched envelope flutter is driving mechanism and is much smaller for high occupancy solenoids mitigating such processes

$\eta = 0.1$



$\eta = 0.75$



Part II: Centroid Ocillations and Steering in Solenoidal Transport

Good News:

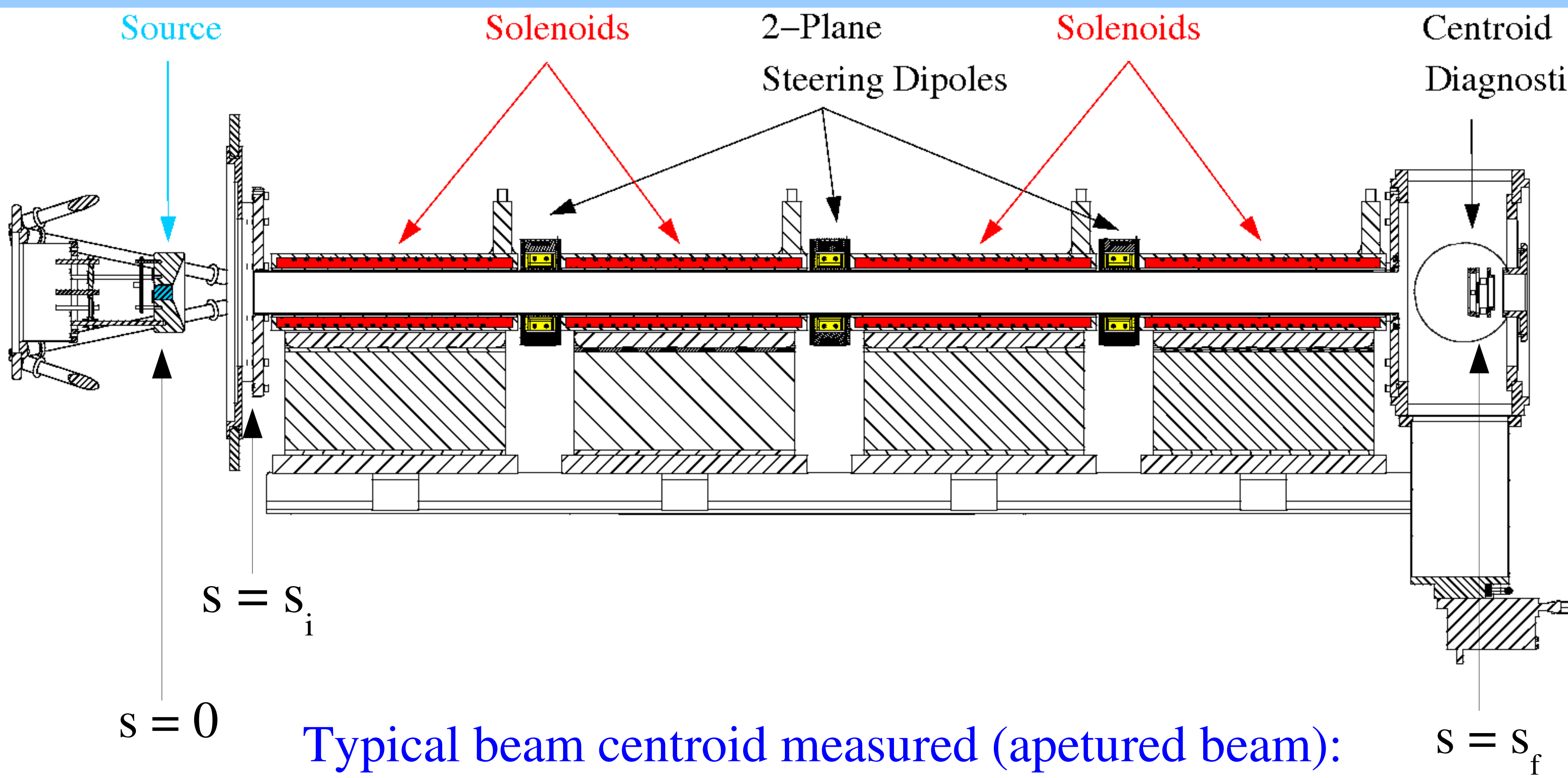
Space-charge related beam stability issues look relatively benign for solenoids relative to quadrupole transport

Question:

What about maintaining centroid control to enable precise focusing on small targets and to mitigate non-ideal effects in transport?

Historically this has been a problem for solenoids. Can theory help?

Solenoid Transport Lattice in the Neutralized Drift Compression Experiment (NDCX):



Typical beam centroid measured (apetured beam):

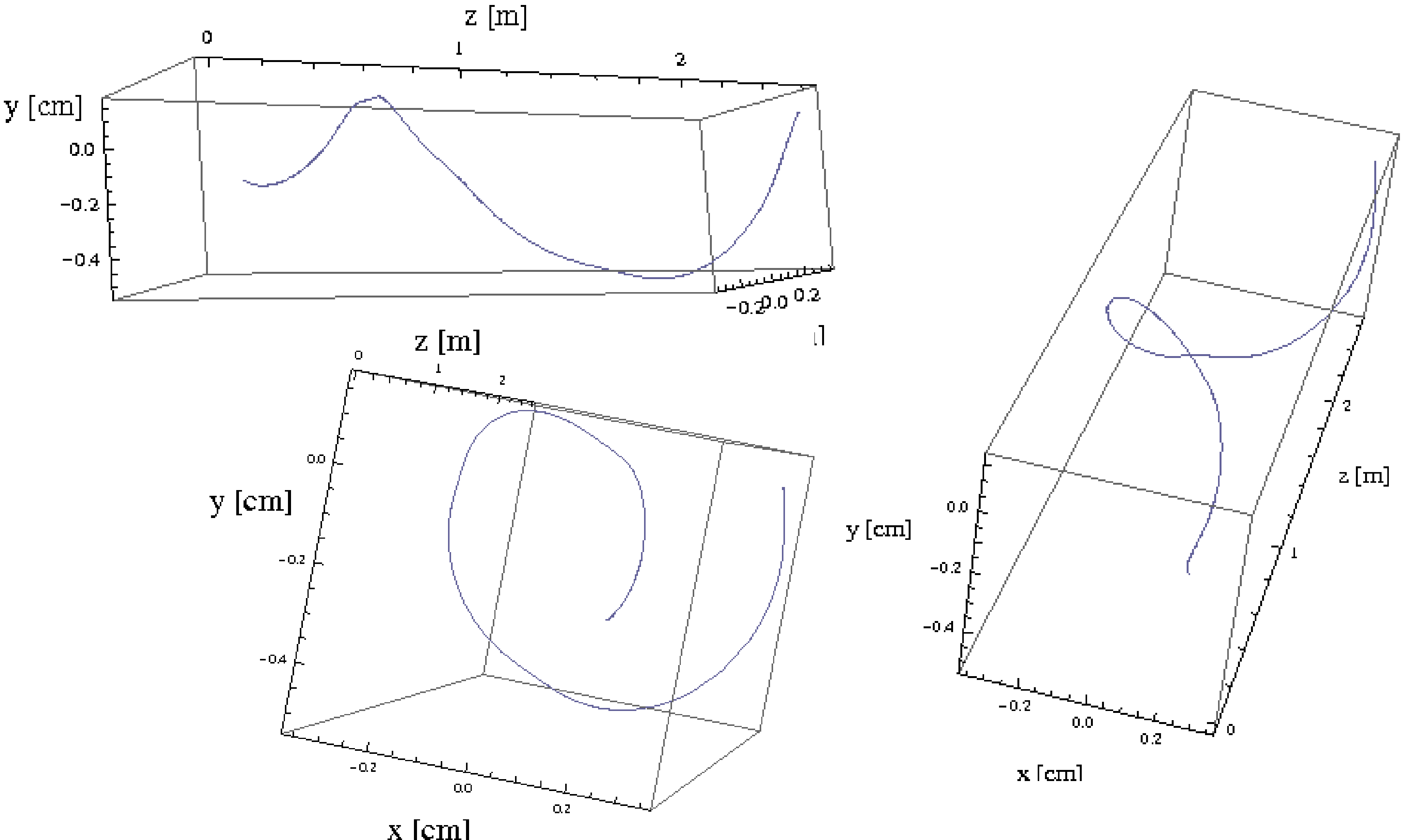
$\langle x \rangle_\perp$ = -5.81 mm $\quad$ $\langle y \rangle_\perp$ = -2.77 mm

$\langle x' \rangle_\perp$ = 2.24 mrad $\quad$ $\langle y' \rangle_\perp$ = 3.37 mrad

Desirable to correct centroid for improved transport and target experiments

Typical centroid orbit in lattice:

- Exhibits *x*-*y* coupling with Larmor rotation
- Driving terms present from solenoid mechanical misalignments



Need to steer orbit back to axis using fixed orientation dipole steering

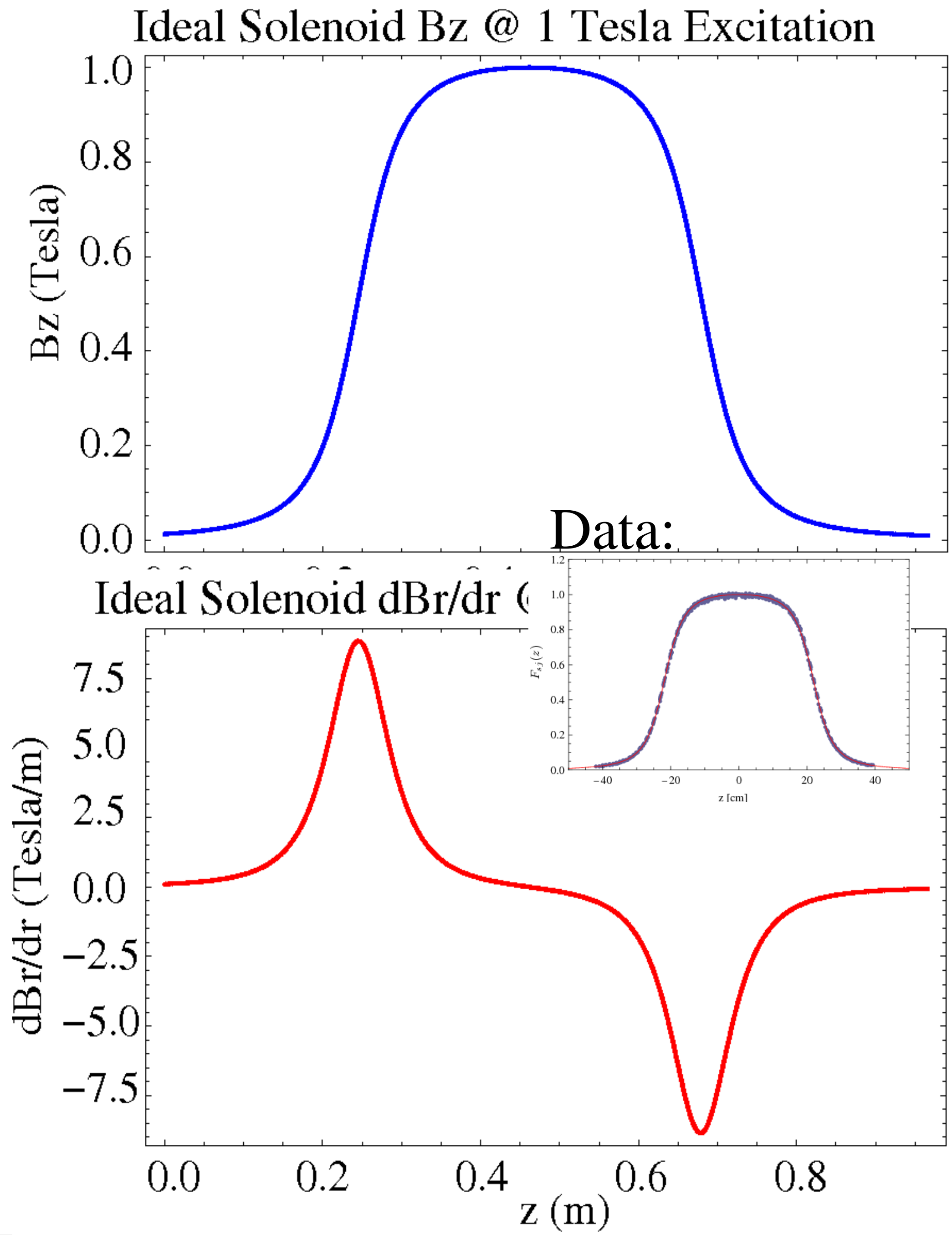
Magnetic Field of “Ideal” Solenoid

Ideal Solenoid Field:

$$B_x(\mathbf{x}) = -\frac{1}{2}\frac{\partial B_{z0}(z)}{\partial z}x$$

$$B_y(\mathbf{x}) = -\frac{1}{2}\frac{\partial B_{z0}(z)}{\partial z}y$$

$$B_z(\mathbf{x}) = B_{z0}(z)$$



NDCX Solenoid Parameters:

Length = 433.1 cm

Radius = 50.8 cm

B_{max} < 3 Tesla

Solenoid fringe function

Linearly Superimpose Solenoids:

$$B_z(s) = \sum_{j=1}^{N_s} B_{sj} F_{sj}(s - s_j)$$

$j =$ solenoid index

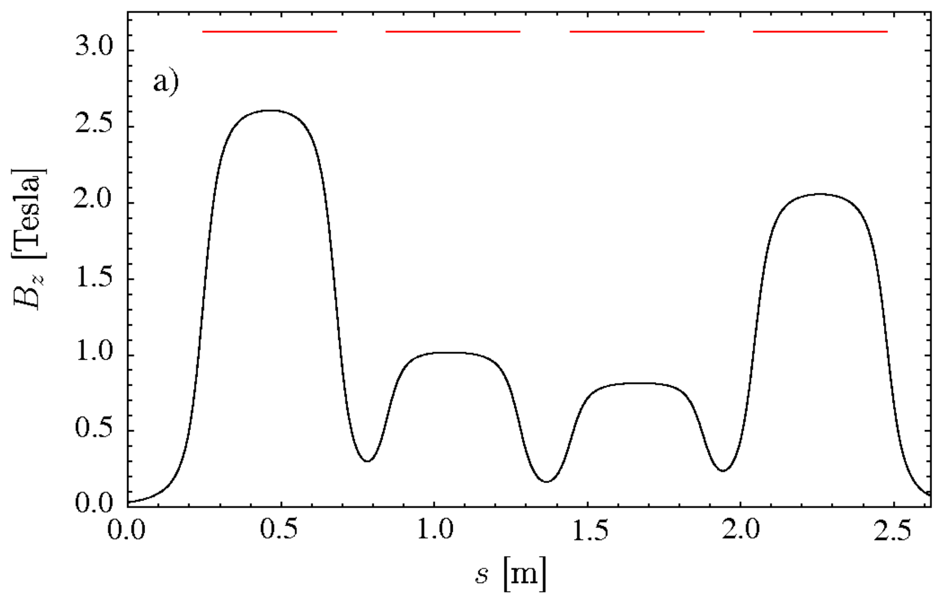
$s_j =$ center jth solenoid

$F_{sj}(s) =$ solenoid Fringe Function

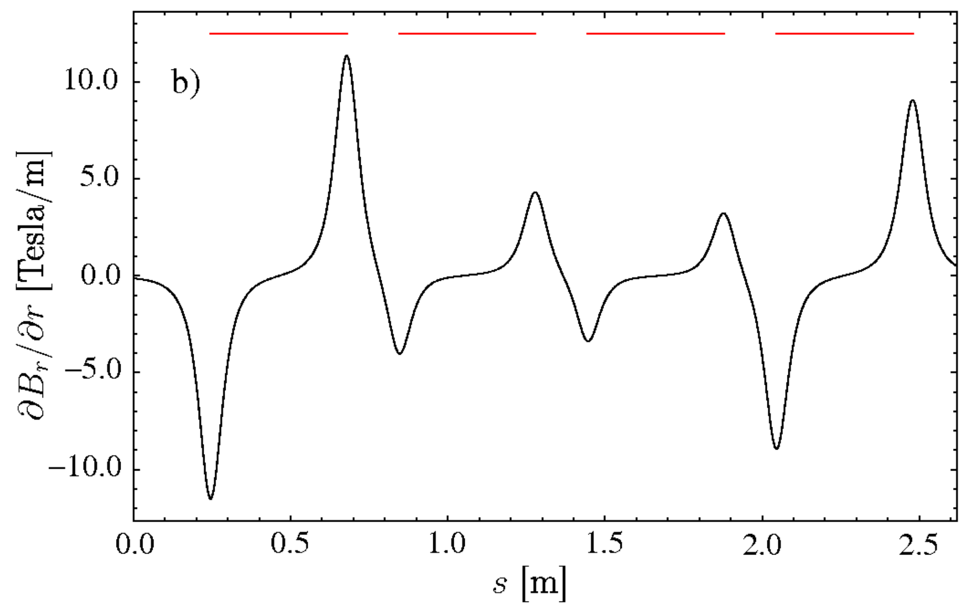
$F_{sj}(0) = 1$ norm

$B_{sj} =$ Peak Field

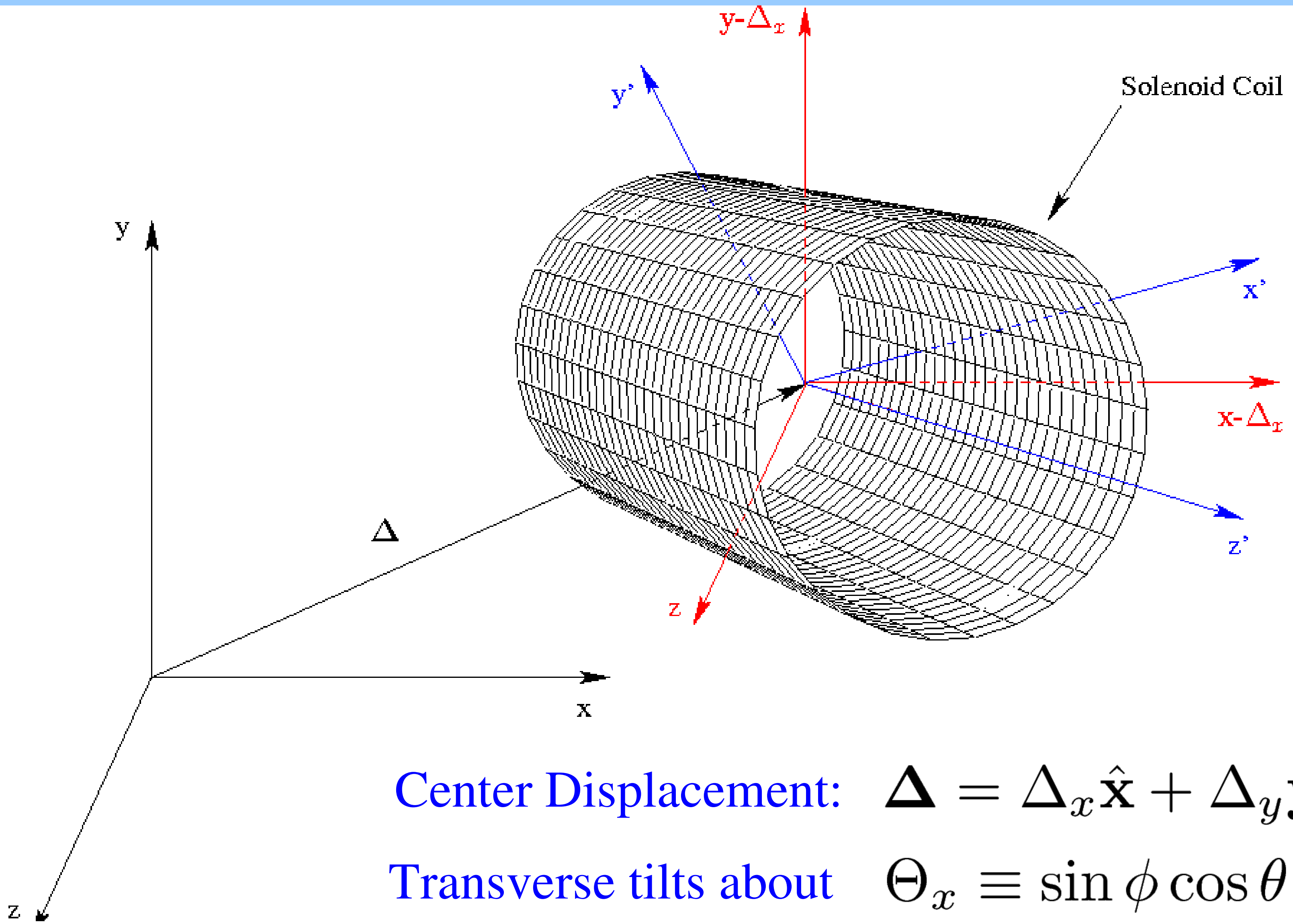
Axial Field:



Transverse Field:



Take an ideal solenoid and then displace the center and tilt about the axis of symmetry to model misalignments



Center Displacement: $\boldsymbol{\Delta} = \Delta_x \hat{\mathbf{x}} + \Delta_y \hat{\mathbf{y}} + \Delta_z \hat{\mathbf{z}}$

Transverse tilts about Displaced center:

$$\Theta_x \equiv \sin\phi\cos\theta \simeq \phi\cos\theta$$

$$\Theta_y \equiv \sin\phi\sin\theta \simeq \phi\sin\theta$$

Leading order magnetic field of misaligned solenoid

Displacement and rotational misalignments:

Center Displacements:

$$\Delta_x, \ \Delta_y, \ \Delta_z$$

$$\frac{|\Delta_x|}{R}, \frac{|\Delta_y|}{R}, \frac{|\Delta_z|}{\ell} \ll 1$$

Rotations (Tilts) of Centerline:

$$\Theta_x, \Theta_y$$

$$\Theta_x, \Theta_y \ll 1$$

Produce bending dipole terms to leading, linear optic order:

Ideal | Misaligned

$$B_x = -\frac{1}{2}\frac{\partial B_{z0}(z)}{\partial z}x + \frac{1}{2}\frac{\partial B_{z0}(z)}{\partial z}[\Delta_x + \Theta_x z] + B_{z0}(z)\Theta_x$$

$$B_y = -\frac{1}{2}\frac{\partial B_{z0}(z)}{\partial z}y + \frac{1}{2}\frac{\partial B_{z0}(z)}{\partial z}[\Delta_y + \Theta_y z] + B_{z0}(z)\Theta_y$$

$$B_z = B_{z0}(z) + \frac{\partial B_{z0}(z)}{\partial z}\Delta_z$$

Magnetic field of steering dipoles

Model as ideal uniform bending fields with axial fringe

$$\boxed{\begin{aligned} B_x(\mathbf{x}) &= B_x F_x(z) \\ B_y(\mathbf{x}) &= B_y F_y(z) \end{aligned}}$$

$$B_x = \text{const}$$

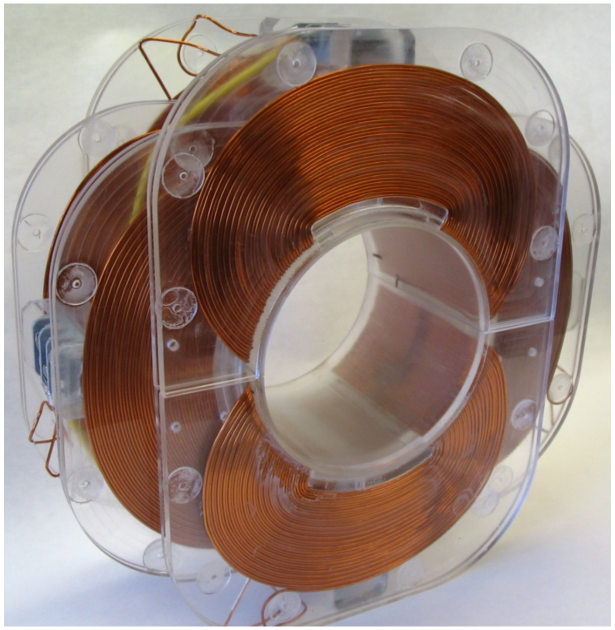
$$F_x(z) = \text{Fringe Function}$$

$$F_x(0) = 1$$

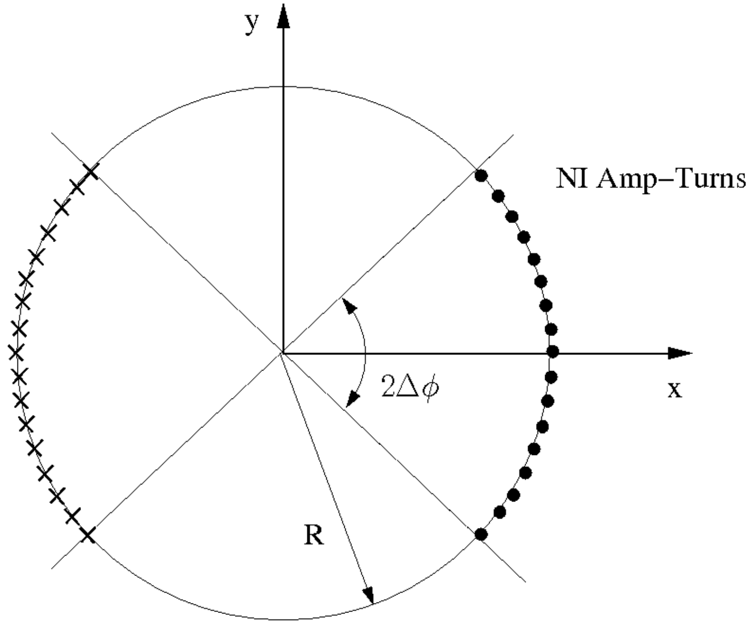
- Misalignment and nonlinear effects neglected since correction is small

In NDCX crossed field dipole correctors are employed

3D Geometry



2D Geometry (*y*-field = *x*-bend)



Transverse Centroid Equations of Motion: Basic Model

Notation:

$$x \equiv \langle x \rangle_\perp \qquad y \equiv \langle y \rangle_\perp$$

$$x' \equiv \langle x' \rangle_\perp \qquad y' \equiv \langle y' \rangle_\perp$$

$$' \equiv \frac{d}{ds}$$

$$[B\rho] \equiv \frac{m\gamma_b\beta_b c}{q}$$ = Rigidity

Equations of Motion:

$$x'' = -\frac{B_y}{[B\rho]} + \frac{B_z}{[B\rho]}y'$$

$$y'' = \frac{B_x}{[B\rho]} - \frac{B_z}{[B\rho]}x'$$

Transverse Centroid Equations of Motion: fully expressed

Use complex coordinates:

$$\underline{z} = x + iy \qquad \underline{z}' = x' + iy' \qquad i \equiv \sqrt{-1}$$

and linearly superimpose solenoid (with errors) and steering fields:

$$\underline{z}'' + i\left(\sum_{j=1}^{N_s} \frac{B_{sj}}{[B\rho]} F_{sj}\right)\underline{z}' + i\left(\sum_{j=1}^{N_s} \frac{B_{sj}}{2[B\rho]} F'_{sj}\right)\underline{z} =$$

$$i\sum_{j=1}^{N_s}\left[\frac{B_{sj}}{2[B\rho]} F'_{sj}\underline{\Delta}_j + \frac{B_{sj}}{2[B\rho]} F'_{sj}\underline{\Theta}_j(s - s_j) + \frac{B_{sj}}{[B\rho]} F_{sj}\underline{\Theta}_j\right]$$

$$+ i\sum_{j=1}^{N_x} \frac{B_{xj}}{[B\rho]} F_{xj} - \sum_{j=1}^{N_y} \frac{B_{yj}}{[B\rho]} F_{yj}$$

Complex transverse misalignments of *j*th solenoid

$\underline{\Delta}_j \equiv \Delta_{xj} + i\Delta_{yj}$ Center Displacement

$\underline{\Theta}_j \equiv \Theta_{xj} + i\Theta_{yj}$ Centerline Rotation

- No dependence to linear order on *z*-displacement Δ_{zj}

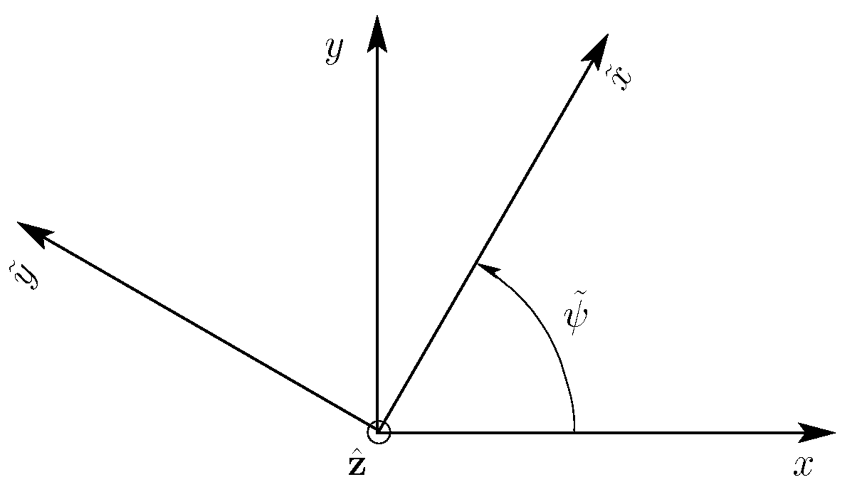
How to solve this mess!

Step 1: Transform particle phase-space to a local rotating "Larmor" frame to remove x-y coupling in the absence of alignment errors and steering

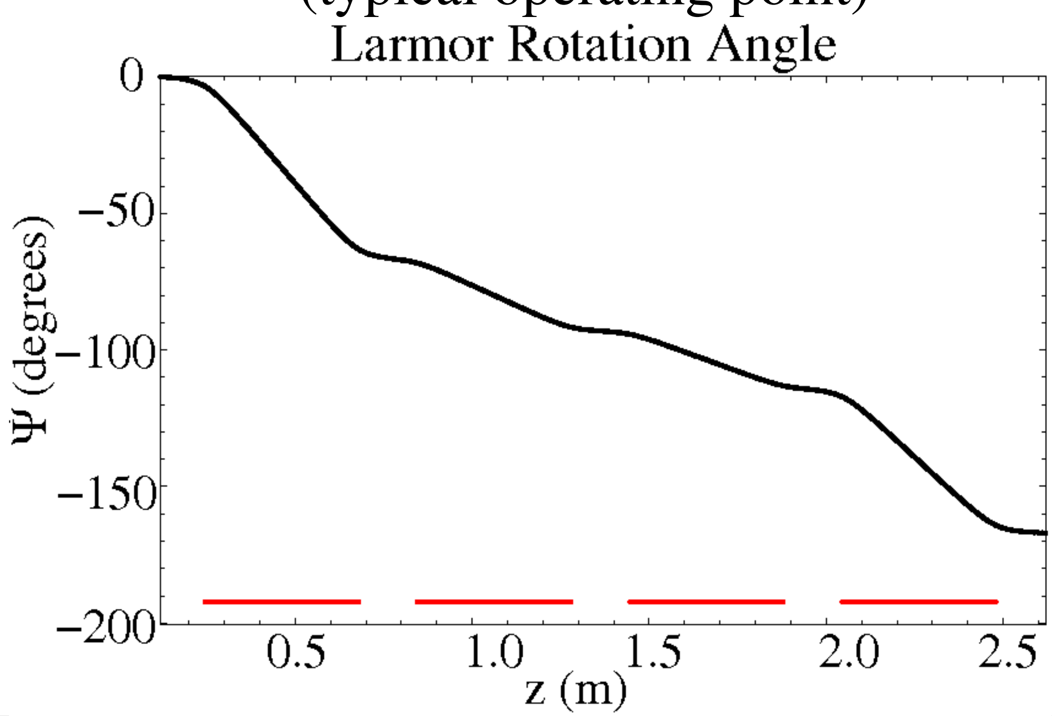
- Quantities with "~" refer to Larmor frame

$$\underline{\tilde{z}} = \underline{z} e^{-i\tilde{\psi}}$$
$$\underline{\tilde{z}}' = \left(\underline{z}' - i\tilde{\psi}'\underline{z}\right) e^{-i\tilde{\psi}}$$

$$\tilde{\psi}(s) = -\frac{1}{2}\sum_{j=1}^{N_s} \frac{B_{sj}}{[B\rho]} \int_{s_i}^{s} d\bar{s}\; F_{sj}(\bar{s} - s_j)$$



(typical operating point)



Step 2: Exploit an analogy to the dispersion function for analysis of "off" momentum in single particle orbit

$$\boxed{x''(s) + \kappa(s)x(s) = \frac{\delta}{R(s)}}$$

$\delta =$ Fractional Momentum Error

$R(s) =$ Radius of Dipole Bend in Latice

Expand: $x \equiv x_h + x_p$

1) homogeneous solution: x_h

$$\boxed{x_h(s)'' + \kappa(s)x_h(s) = 0}$$

Usual Hill's Equation

2) particular solution periodic with lattice: x_p

$$x_p(s) = \delta \cdot D(s)$$

$$D''(s) + \kappa(s)D(s) = \frac{1}{R(s)}$$

$$D(s + \text{Period}) = D(s)$$

Usual Dispersion Function

Property of Lattice Only

Obtain Expanded Centroid Solution

Total Solution = Homogeneous Solution (ideal, aligned response)
+ Particular Solutions (misalignment + steering response)

$$\underline{\tilde{z}}(s) = \underline{\tilde{z}}(s_i)\tilde{C}(s|s_i) + \underline{\tilde{z}}'(s_i)\tilde{S}(s|s_i)$$

Solution without alignment errors from initial condition

$$+ \sum_{j=1}^{N_s} [\underline{\Delta}_j \underline{\tilde{D}}_j(s) + \underline{\Theta}_j \underline{\tilde{R}}_j(s)]$$

Solenoid alignment error terms

$$\underline{\Delta}_j \equiv \Delta_{xj} + i\Delta_{yj}$$
$$\underline{\Theta}_j \equiv \Theta_{xj} + i\Theta_{yj}$$

Complex transverse misalignments of *j*th solenoid

$$+ \sum_{j=1}^{N_x} \frac{B_{xj}}{[B\rho]} \underline{\tilde{D}}_{xj}(s) + \sum_{j=1}^{N_y} \frac{B_{yj}}{[B\rho]} \underline{\tilde{D}}_{yj}(s)$$

Steering terms

Homogeneous Solution: Cosine and Sine-like Functions

Cosine and Sine-like functions are real-valued and obey the Larmor-frame Hill's Equation:

$$\boxed{\begin{aligned} \tilde{C}''(s|s_i) + \tilde{\kappa}(s)\tilde{C}(s|s_i) = 0 \\ \tilde{S}''(s|s_i) + \tilde{\kappa}(s)\tilde{S}(s|s_i) = 0 \end{aligned}}$$

$$\tilde{\kappa}(s) \equiv \frac{1}{4}\left[\sum_{j=1}^{N_s} \frac{B_{sj}}{[B\rho]} F_{sj}(s - s_j)\right]^2$$

Focusing strength in the Larmor frame

Satisfying the “cosine-” and “sine-like” initial conditions:

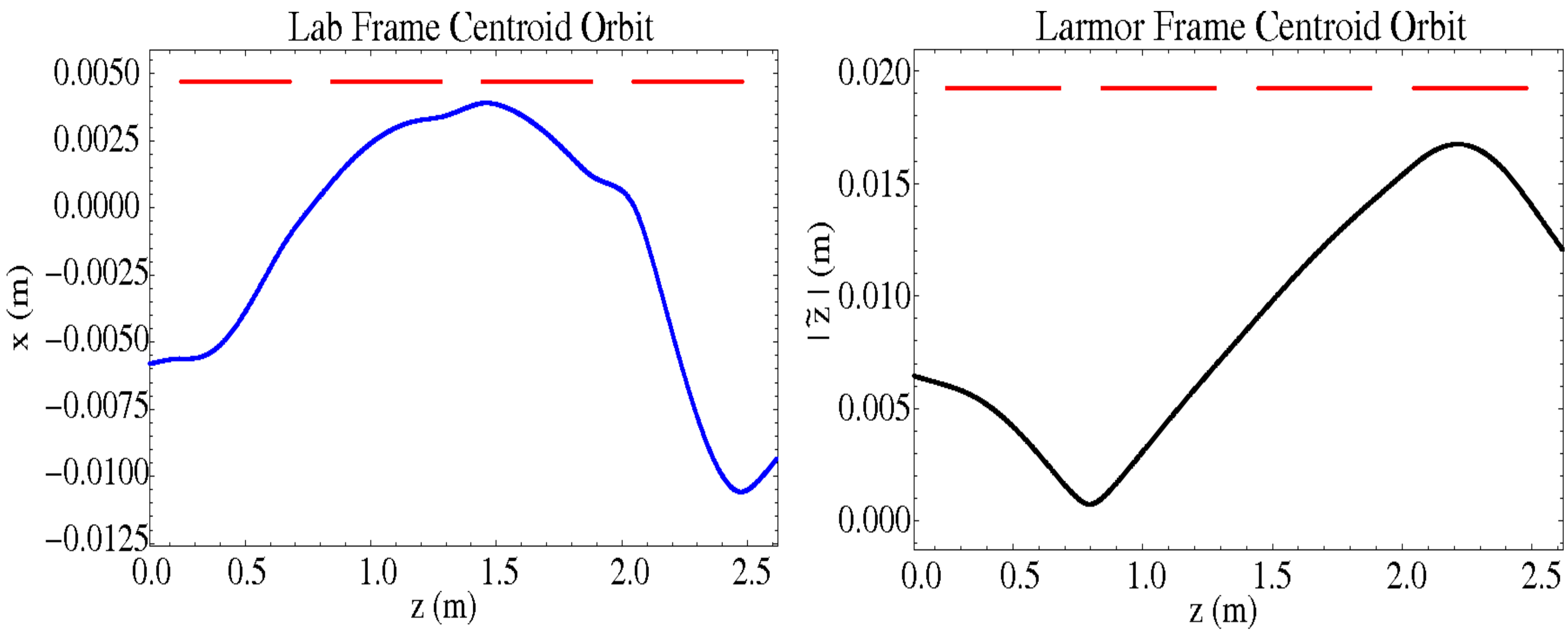
$$\tilde{C}(s_i|s_i) = 1 \qquad \tilde{S}(s_i|s_i) = 0$$

$$\tilde{C}'(s_i|s_i) = 0 \qquad \tilde{S}'(s_i|s_i) = 1$$

Gives the Larmor-frame orbit from initial conditions in the ideal system

- *x*-*y* plane decoupled with usual Courant-Snyder invariants etc.

Centroid Orbit for an Aligned Lattice



Shown for "plausible" initial conditions
of centroid emerging from injector:

$$x_i, y_i \approx \ 1 \text{ mm}$$
$$x'_i, y'_i \approx \ 1 \text{ mrad}$$

Larmor transformation simplifies expression of centroid oscillations

Particular Solution: complex-valued *Alignment Functions*

Driven Hill's equations solved with homogeneous initial conditions:

Displacement Function:

$$\boxed{\underline{\tilde{D}}_j'' + \tilde{\kappa}\underline{\tilde{D}}_j = \frac{i}{2}\frac{B_{sj}}{[B\rho]}F_{sj}' e^{-i\tilde{\psi}}}$$

$$\underline{\tilde{D}}_j(s_i) = 0 = \underline{\tilde{D}}_j'(s_i)$$

LHS: Hill's Equation

RHS: Driving dipole terms generated by misalignment

Rotation Function

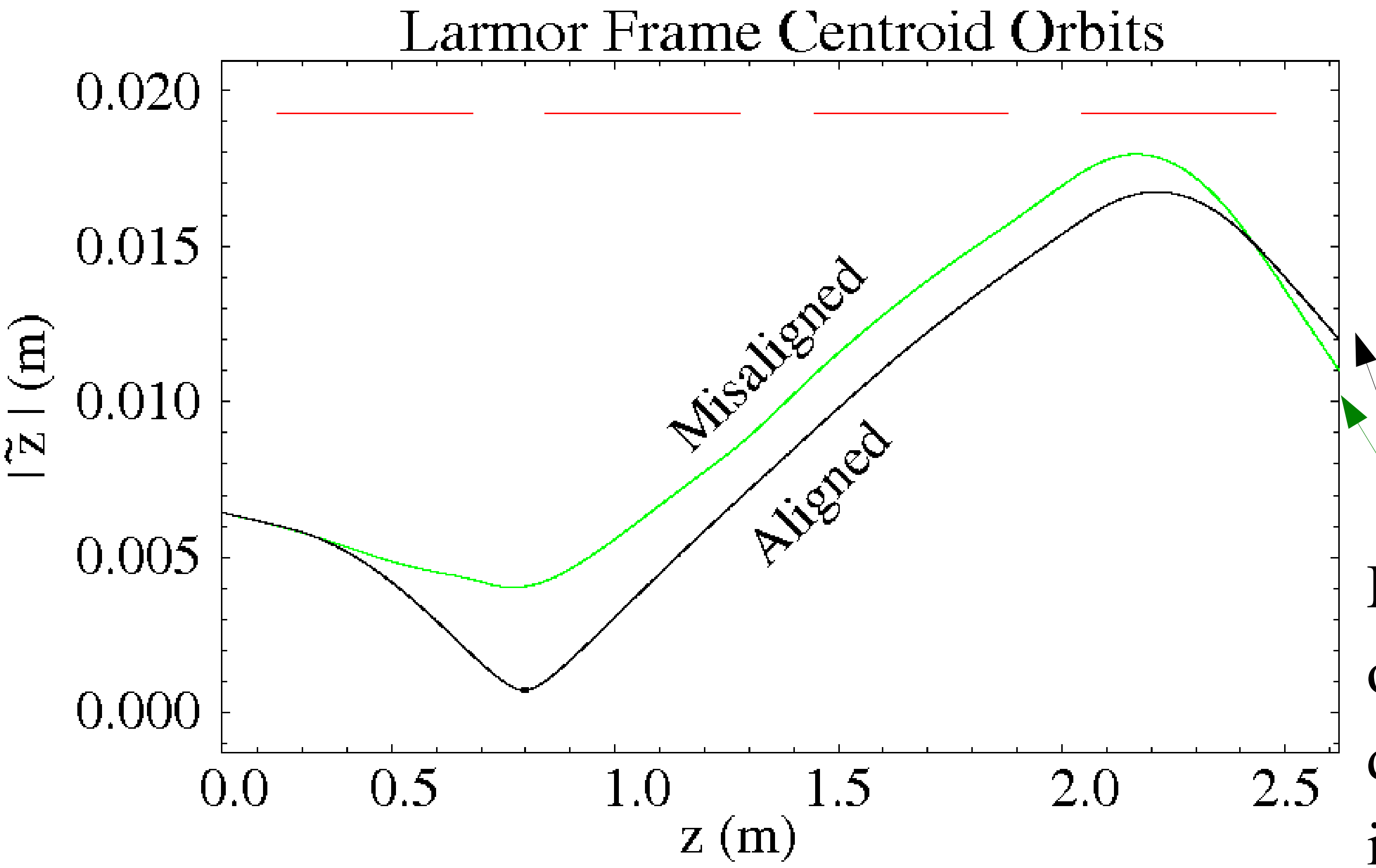
$$\boxed{\underline{\tilde{R}}_j'' + \tilde{\kappa}\underline{\tilde{R}}_j = \frac{i}{2}\frac{B_{sj}}{[B\rho]}F_{sj}'(s - s_j)e^{-i\tilde{\psi}} + i\frac{B_{sj}}{[B\rho]}F_{sj}e^{-i\tilde{\psi}}}$$

$$\underline{\tilde{R}}_j(s_i) = 0 = \underline{\tilde{R}}_j'(s_i)$$

Gives change in Larmor-frame orbit due to mechanical misalignments

- Dipole terms driven by misalignments have amplitudes scaled out
- $\underline{\tilde{D}}_j$, $\underline{\tilde{R}}_j$ functions of the *ideal*, aligned lattice

Centroid Orbit for a Misaligned Lattice



Misalignments of solenoids cause deviations from ideal positioning.

Shown for a set of solenoid randomly set misalignments with:

$$\Delta_{xj}, \Delta_{yj} \approx 1 \text{ mm}$$

$$\Theta_{xj}, \Theta_{yj} \approx 1 \text{ mrad}$$

Particular Solution: Bending Functions

Driven Hill's equations solved with homogeneous initial conditions:

$$\underline{\tilde{B}}''_{xj} + \tilde{\kappa}\underline{\tilde{B}}_{xj} = iF_{xj}e^{-i\tilde{\psi}},$$
$$\underline{\tilde{B}}''_{yj} + \tilde{\kappa}\underline{\tilde{B}}_{yj} = -F_{yj}e^{-i\tilde{\psi}},$$

LHS: Hill's Equation

RHS: Driving terms generated by steering dipoles

$$\underline{\tilde{B}}_{xj}(s_i) = 0 = \underline{\tilde{B}}'_{xj}(s_i)$$

$$\underline{\tilde{B}}_{yj}(s_i) = 0 = \underline{\tilde{B}}'_{yj}(s_i)$$

Gives change in Larmor-frame orbit due to steering dipoles

- Steering dipole terms have amplitudes scaled out
- $\underline{\tilde{B}}_{xj}, \; \underline{\tilde{B}}_{yj}$ functions of the *ideal*, aligned lattice

Application: Statistical Analysis of Misalignments

- Use orbit expansion derived to efficiently calculate statistical properties of an ensemble of errors. Show example with:

  -- 10,000 independent error sets

  -- Uniformly distributed up to cutoff amplitudes

  -- *x*- and *y*-planes same up to statistical errors: show only *x*-plane

- Misalignments can come in 2 flavors:

  Error of initial centroid out of gun:

  coordinate and angles take:

  $$|x_i| < 2 \text{ mm} \qquad |x'_i| < 5 \text{ mrad}$$

  Alignment of individual solenoids:

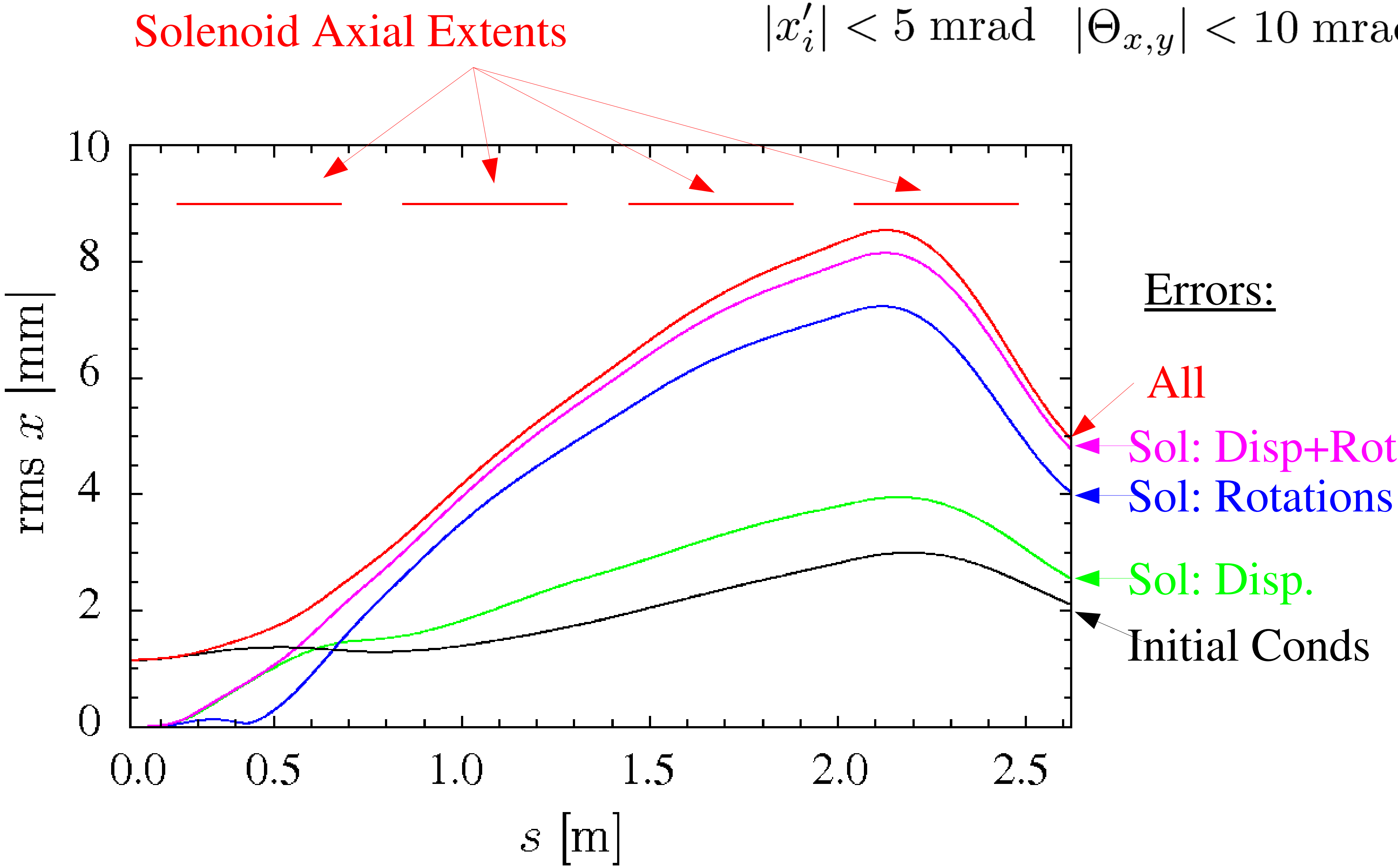
  transverse displacements and rotations:

  $$|\Delta_{x,y}| < 3 \text{ mm} \qquad |\Theta_{x,y}| < 10 \text{ mrad}$$

rms centroid coordinate

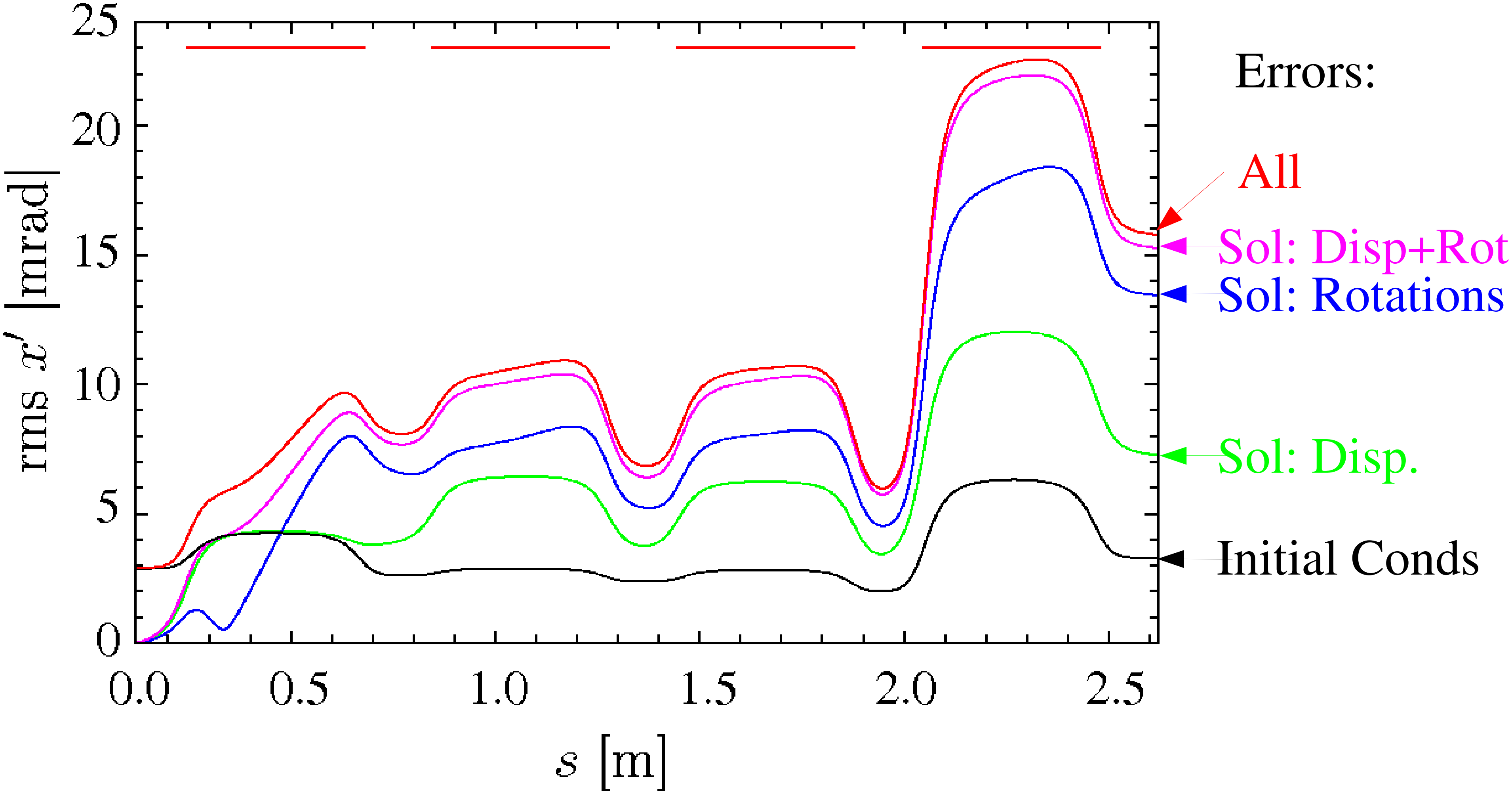
$|x_i| < 2$ mm $\quad |\Delta_{x,y}| < 3$ mm

$|x_i'| < 5$ mrad $\quad |\Theta_{x,y}| < 10$ mrad



rms centroid angle

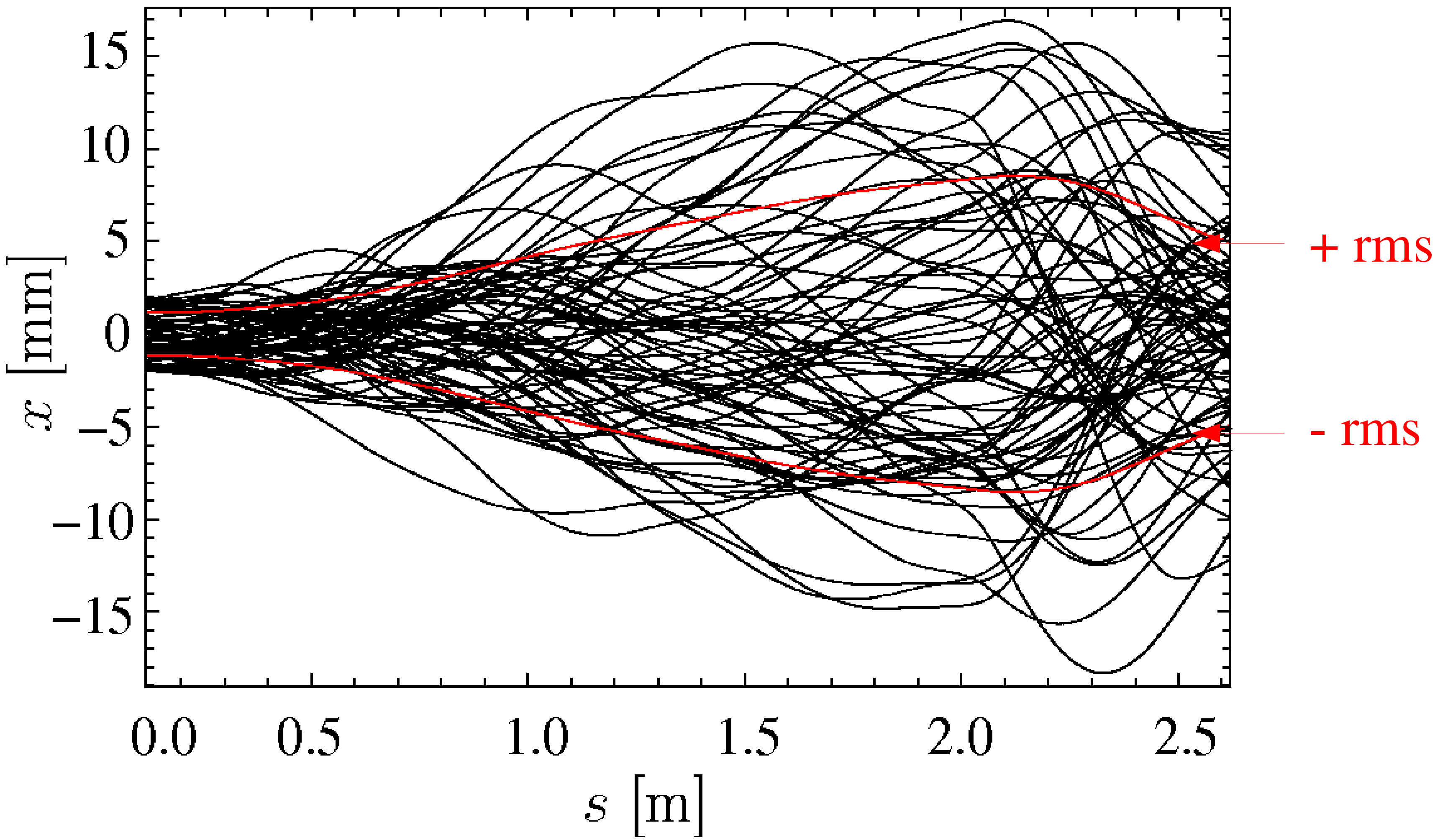
$$|x_i| < 2 \text{ mm} \quad |\Delta_{x,y}| < 3 \text{ mm}$$
$$|x'_i| < 5 \text{ mrad} \quad |\Theta_{x,y}| < 10 \text{ mrad}$$



rms centroid coordinate + orbit bundle

Bundle of 75 orbits in ensemble

$$|x_i| < 2 \text{ mm} \qquad |\Delta_{x,y}| < 3 \text{ mm}$$
$$|x_i'| < 5 \text{ mrad} \qquad |\Theta_{x,y}| < 10 \text{ mrad}$$



Application: Optimal Beam Steering

Many specific steering systems possible that theory can be applied

- Assume (as in NDCX) two crossed field dipoles used to steer

Matrix of Bending Functions at measurement location **[B]**

Bend Field Strengths $\vec{B}$

$$\begin{bmatrix} \mathrm{Re}[\underline{\tilde{B}}_{x1m}] & \mathrm{Re}[\underline{\tilde{B}}_{y1m}] & \mathrm{Re}[\underline{\tilde{B}}_{x2m}] & \mathrm{Re}[\underline{\tilde{B}}_{y2m}] \\ \mathrm{Re}[\underline{\tilde{B}}'_{x1m}] & \mathrm{Re}[\underline{\tilde{B}}'_{y1m}] & \mathrm{Re}[\underline{\tilde{B}}'_{x2m}] & \mathrm{Re}[\underline{\tilde{B}}'_{y2m}] \\ \mathrm{Im}[\underline{\tilde{B}}_{x1m}] & \mathrm{Im}[\underline{\tilde{B}}_{y1m}] & \mathrm{Im}[\underline{\tilde{B}}_{x2m}] & \mathrm{Im}[\underline{\tilde{B}}_{y2m}] \\ \mathrm{Im}[\underline{\tilde{B}}'_{x1m}] & \mathrm{Im}[\underline{\tilde{B}}'_{y1m}] & \mathrm{Im}[\underline{\tilde{B}}'_{x2m}] & \mathrm{Im}[\underline{\tilde{B}}'_{y2m}] \end{bmatrix} \cdot \begin{bmatrix} B_{x1} \\ B_{y1} \\ B_{x2} \\ B_{y2} \end{bmatrix}$$

$$= -[B\rho] \begin{bmatrix} \mathrm{Re}[\underline{\tilde{z}}_t - \underline{\tilde{z}}_m] \\ \mathrm{Re}[\underline{\tilde{z}}'_t - \underline{\tilde{z}}'_m] \\ \mathrm{Im}[\underline{\tilde{z}}_t - \underline{\tilde{z}}_m] \\ \mathrm{Im}[\underline{\tilde{z}}'_t - \underline{\tilde{z}}'_m] \end{bmatrix}$$

Target minus measured centroid coordinates (Larmor transformed) ΔZ

$$\begin{aligned} \underline{\tilde{B}}_{x1m} &\equiv \underline{\tilde{B}}_{x1}(s_m) \\ &= x\text{-plane bending function at measurement } s = s_m \end{aligned}$$

$$[B] \cdot \vec{B} = -[B\rho]\Delta Z$$

Solve for steering fields as:

$$\vec{B} = -[B\rho][B]^{-1} \cdot \Delta Z$$

- 5x times less work (1 vs 5 full phase-space measurements needed) than “pure” experimental method of accumulating a deviation Jacobian

Jacobian-based pure experimental correction procedure can be applied

Two crossed dipole example: $x \equiv \langle x \rangle_\perp$ etc.

$$\begin{bmatrix} \frac{\delta x}{\delta I_{2x}} & \frac{\delta x}{\delta I_{2y}} & \frac{\delta x}{\delta I_{3x}} & \frac{\delta x}{\delta I_{3y}} \\ \frac{\delta x'}{\delta I_{2x}} & \frac{\delta x'}{\delta I_{2y}} & \frac{\delta x'}{\delta I_{3x}} & \frac{\delta x'}{\delta I_{3y}} \\ \frac{\delta y}{\delta I_{2x}} & \frac{\delta y}{\delta I_{2y}} & \frac{\delta y}{\delta I_{3x}} & \frac{\delta y}{\delta I_{3y}} \\ \frac{\delta y'}{\delta I_{2x}} & \frac{\delta y'}{\delta I_{2y}} & \frac{\delta y'}{\delta I_{3x}} & \frac{\delta y'}{\delta I_{3y}} \end{bmatrix} \cdot \begin{bmatrix} \delta I_{2x} \\ \delta I_{2y} \\ \delta I_{3x} \\ \delta I_{3y} \end{bmatrix} = \cancelto{0}{\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_T} - \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_0$$

Response Matrix [D]

Deviation Currents $\vec{\delta I}$

Coords: Target value - Nominal value $\vec{x}_0$

$$\boxed{\vec{\delta I} = -[D]^{-1} \cdot \vec{x}_0}$$

Advantages:

- Takes into account all effects (caution: even with wrong interpretation)

Disadvantages:

- Min 5x (6x to verify) measures of full centroid phase space needed
  - More if nonlinear: iterate corrections to reduce amplitudes
- Must repeat every time machine operating point is changed

Application: Calculation of lattice misalignment parameters

Measure enough operating points to constrain all misalignment parameters and use theory to relate to optical response:

$$\underline{\tilde{Z}} = \underline{\tilde{A}} \cdot \underline{D}$$

$\underline{\tilde{Z}} \equiv$ Centroid measurement data of *k* operating points of lattice: (*2k*)

$\underline{\tilde{A}} \equiv$ Principal orbit and alignment functions evaluated at the measurement plane for the *k* operating points: (*2k*) x (*2Ns* + *2*)

$\underline{D} \equiv$ Unknown initial centroid and solenoid misalignment parameters: (*2Ns* +*2*)

$$\begin{pmatrix} \underline{\tilde{z}}^1_m \\ \underline{\tilde{z}}'^1_m \\ \underline{\tilde{z}}^2_m \\ \underline{\tilde{z}}'^2_m \\ \vdots \\ \underline{\tilde{z}}^k_m \\ \underline{\tilde{z}}'^k_m \end{pmatrix} = \begin{pmatrix} \tilde{C}^1_m & \tilde{S}^1_m & \underline{\tilde{D}}^1_{1m} & \underline{\tilde{D}}^1_{2m} & \cdots & \underline{\tilde{D}}^1_{N_s m} & \underline{\tilde{R}}^1_{1m} & \underline{\tilde{R}}^1_{2m} & \cdots & \underline{\tilde{R}}^1_{N_s m} \\ \tilde{C}'^1_m & \tilde{S}'^1_m & \underline{\tilde{D}}'^1_{1m} & \underline{\tilde{D}}'^1_{2m} & \cdots & \underline{\tilde{D}}'^1_{N_s m} & \underline{\tilde{R}}'^1_{1m} & \underline{\tilde{R}}'^1_{2m} & \cdots & \underline{\tilde{R}}^1_{N_s m} \\ \tilde{C}^2_m & \tilde{S}^2_m & \underline{\tilde{D}}^2_{1m} & \underline{\tilde{D}}^2_{2m} & \cdots & \underline{\tilde{D}}^2_{N_s m} & \underline{\tilde{R}}^2_{1m} & \underline{\tilde{R}}^2_{2m} & \cdots & \underline{\tilde{R}}^2_{N_s m} \\ \tilde{C}'^2_m & \tilde{S}'^2_m & \underline{\tilde{D}}'^2_{1m} & \underline{\tilde{D}}'^2_{2m} & \cdots & \underline{\tilde{D}}'^2_{N_s m} & \underline{\tilde{R}}'^2_{1m} & \underline{\tilde{R}}'^2_{2m} & \cdots & \underline{\tilde{R}}^2_{N_s m} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \tilde{C}^k_m & \tilde{S}^k_m & \underline{\tilde{D}}^k_{1m} & \underline{\tilde{D}}^k_{2m} & \cdots & \underline{\tilde{D}}^k_{N_s m} & \underline{\tilde{R}}^k_{1m} & \underline{\tilde{R}}^k_{2m} & \cdots & \underline{\tilde{R}}^k_{N_s m} \\ \tilde{C}'^k_m & \tilde{S}'^k_m & \underline{\tilde{D}}'^k_{1m} & \underline{\tilde{D}}'^k_{2m} & \cdots & \underline{\tilde{D}}'^k_{N_s m} & \underline{\tilde{R}}'^k_{1m} & \underline{\tilde{R}}'^k_{2m} & \cdots & \underline{\tilde{R}}^k_{N_s m} \end{pmatrix} \cdot \begin{pmatrix} \underline{\tilde{z}}_i \\ \underline{\tilde{z}}'_i \\ \underline{\Delta}_1 \\ \underline{\Delta}_2 \\ \vdots \\ \underline{\Delta}_{N_s} \\ \underline{\Theta}_1 \\ \underline{\Theta}_2 \\ \vdots \\ \underline{\Theta}_{N_s} \end{pmatrix}$$

Solve using over-constrained data sets using SVD methods:

- Answer is then best fit in a least-square sense to data

$$\boxed{\underline{D} = \underline{\tilde{A}}^{-1} \cdot \underline{\tilde{Z}}}$$

Comments:

- Determining actual misalignments is a first and enables:
  - Mechanical correction of misalignments
  - Incorporation of specific misalignments in detailed simulations
- More variables will generally result in worse fit
  - Use pencil beam and turn off solenoids, if possible
- Errors must be studied for high confidence inversion
  - Numerical tests indicate +- 0.5 mm in *x,y* and +- 1 mrad in *x',y'* measurement tolerances allows determination of 2-4 solenoids with initial conditions
- Attempts at calculating errors with lab measurements have failed to date and suggest some inconsistency in measurement of theory

Conclusion: Centroid oscillations and steering in solenoidal transport

A new formulation has been derived to efficiently analyze the linear evolution of the beam centroid

- Larmor frame analysis using an expansion in complex-valued functions that depend only on ideal lattice properties
- Analogous to "Dispersion Function" treatments of momentum spread
  - Functions for each element and more complicated formulation because each element can have different mechanical misalignments

This formulation can be exploited to:

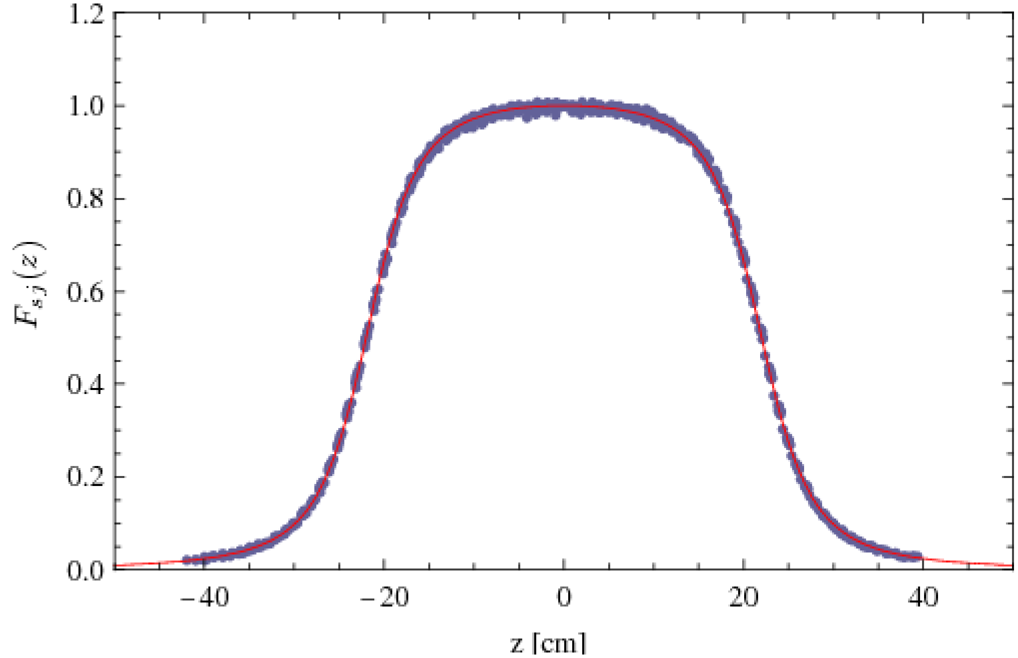
- Efficiently analyze statistical properties of alignment errors to set tolerances
- Optimally steer the beam
- Calculate actual misalignments of lattice from centroid measurements

Applications to the NDCX solenoid transport lattice underway:

- Statistical predictions appear consistent with measurements
- Steering and alignment error inversion are still underway
  - Agreement is improving with measurement refinements
  - Still not good enough at present but promising: Hard to measure mrad angles!

Extra Slides

Field data measured from 4 solenoids of the NDCX lattice and fit to the thin-coil model using nonlinear regression:



~ 800 data points

Solenoid Geometry:

Length Coil = 43.31 cm

Radius Coil (inner) = 5.08 cm

Radius Coil (outer) = 6.60 cm

B_{max} < 3 Tesla

Best Thin-Coil Fit Results:

ℓ Length Coil = 43.685 cm

R Radius Coil (inner) = 6.031 cm

$$F_s(z) = \frac{B_{z0}(z)}{B_{z0}(0)} \propto \frac{1}{2}\left(\frac{\ell/2 - z}{\sqrt{(\ell/2 - z)^2 + R^2}} + \frac{\ell/2 + z}{\sqrt{(\ell/2 + z)^2 + R^2}}\right)$$

Larmor Frame Transformation: Decouples ideal betatron oscillations

Use complex coordinates:

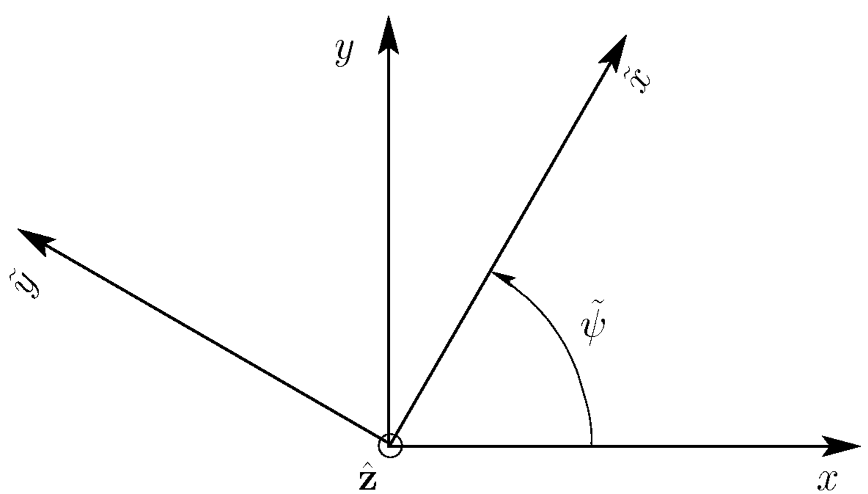
$$\underline{z} = x + iy \qquad \underline{z}' = x' + iy' \qquad i \equiv \sqrt{-1}$$

Transform particle phase-space to a local rotating "Larmor" frame

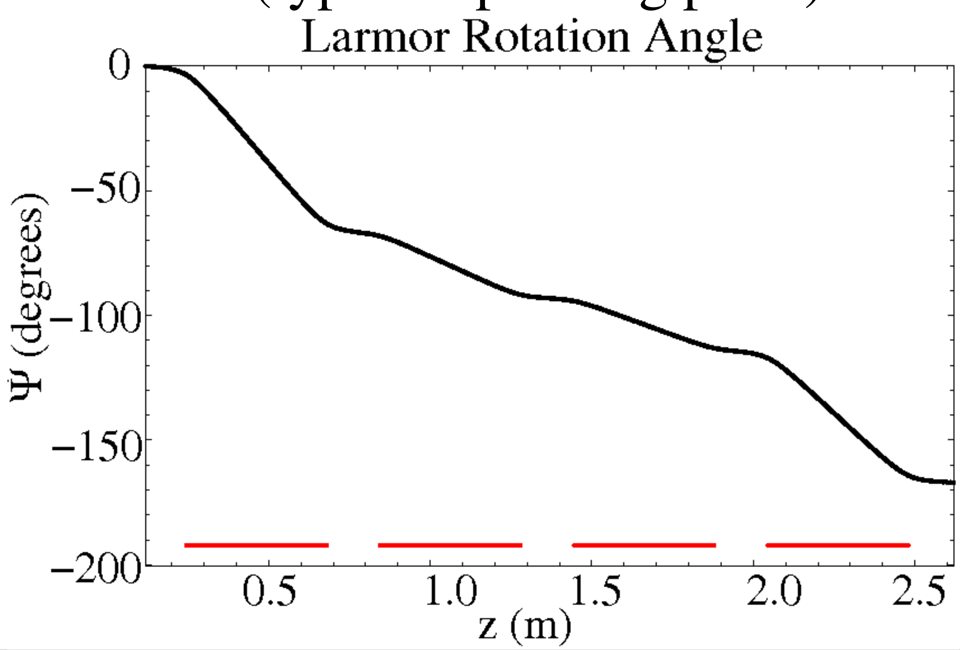
- Quantities with "~" refer to Larmor frame

$$\tilde{\underline{z}} = \underline{z}e^{-i\tilde{\psi}}$$
$$\tilde{\underline{z}}' = \left(\underline{z}' - i\tilde{\psi}'\underline{z}\right) e^{-i\tilde{\psi}}$$

$$\tilde{\psi}(s) = -\frac{1}{2}\sum_{j=1}^{N_s} \frac{B_{sj}}{[B\rho]} \int_{s_i}^{s} d\bar{s}\; F_{sj}(\bar{s} - s_j)$$



(typical operating point)



rms centroid angle + orbit bundle

Bundle of 75 orbits in ensemble

$$|x_i| < 2 \text{ mm} \quad |\Delta_{x,y}| < 3 \text{ mm}$$

$$|x_i'| < 5 \text{ mrad} \quad |\Theta_{x,y}| < 10 \text{ mrad}$$