Constraint on Longitudinal Velocity Spread for Beams Undergoing Longitudinal Compression

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The initial longitudinal temperature of a beam imposes an important limit on the minimum beam length that can be achieved by neutralized compression. Before compression, the initial longitudinal phase-space of an idealized beam might look like the left sketch in Fig. 1, with an initial length L_i and a linear head-to-tail variation Δv_z in the locally averaged longitudinal velocity. At any point z along the beam, there will be some velocity spread δv_z about the average value due to the finite longitudinal temperature of the beam.

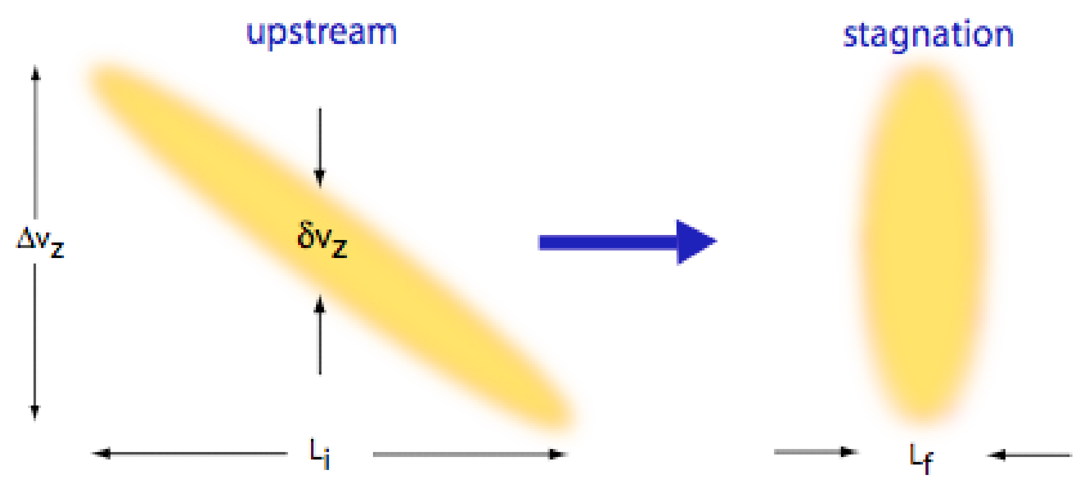

Figure 1: Sketch of a beam longitudinal phase-space to illustrate neutralized compression.

If we assume complete neutralization of the beam space charge and an absence of collisions and external electric fields, the beam phase-space dynamics becomes very simple. Under these conditions, the longitudinal velocity v_z of a beam ion remains constant during compression. Consequently, ions with the same v_z but different longitudinal positions, given by their coordinate z, will move together as the beam propagates through the transport lattice but shift in z relative to particles with different v_z values. If one pictures the beam as a stack of horizontal slices, each with constant v_z, then it is obvious that the phase-space area of the beam, and hence the longitudinal emittance, is constant during compression. It is also evident that the minimum length is just the longitudinal extent of the slice traveling at the average velocity of the whole beam, $<v_z>$. When $\square v_z << \square v_z$, this minimum length L_f is well-approximated by noting that $\square v_z$ bears the same relation to $\square v_z$ as L_f does to L_{i}, giving the approximate relation

$$L_f \approx L_i \, \square v_z / \square v_z. \tag{1}$$

A more careful calculation, assuming that the beam phase-space distribution is initially ellipsoidal in the z - v_z plane, gives the relation

$$(L_i / L_f) - 1 \approx (\square v_z / \square v_z). \tag{2}$$

When $\square v_z$ goes to zero, Eq. (2) gives the correct result, $L_f \sim L_i$, and the expression reduces to Eq. (1) when $\square v_z << \square v_z$. For 100:1 compression and $\square v_z / v_z = 0.1$, Eq. (2) indicates that $\square v_z / <v_z>$ must be 0.001 or less. For a 20-MeV beam, this constraint would limit longitudinal temperature to less than 40 keV. Since we expect a longitudinal temperature after injection on the order of 10 eV, a 40-keV limit before compression leaves a comfortable margin for heating during acceleration.

We should point out that Eq. (2) ignores several effects that could undermine neutralized compression. Any residual electrostatic charge will resist compression and, if the fields are nonlinear, may increase the longitudinal temperature. Scattering due to collisions with the background plasma will increase the temperature by transferring longitudinal energy to the transverse direction, and beam stripping from these collisions will increase the beam charge, possibly requiring a higher plasma density for effective neutralization. Also, the L_f expression assumes that transport is stable during compression. These effects should be analyzed and, if necessary, simulated before we accept Eq. (2) as a plausible estimate of neutralized compression.