ADIABATIC PLASMA LENS – A CURRENT DENSITY BOOSTER

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The adiabatic plasma lens provides a mechanism for increasing the current density at target by up to a factor of 10. The physical setup consists of a tapered insulating tube filled with a gas at <~ 1 torr density. An external discharge (~10kV) initiates a current (~50kA) along the length of the tapered tube. The adiabatically increasing azimuthal magnetic field confines and further reduces the beam size as it traverses down the tube. This tapered tube is located at the downstream end of a high-field focusing solenoid, and immediately upstream of the target.

In this note, we describe the basic physics of the plasma lens, and present an HEDP relevant numerical example at the end.

Charge and current neutralization

The fundamental premise of an adiabatic plasma lens is that the highly conducting medium of the current-carrying channel provides total charge and current neutralizing for the incoming ion beam. The particle dynamics therefore reduces to simple single particle orbits in an azimuthal magnetic field. A typical example would be a fully ionized channel with a temperature of 7eV. The conductivity is $\sigma \sim 10^{14}\ \mathrm{sec}^{-1}$ [units in cgs]. Charge neutralization takes place in a time exceedingly short relative to the pulse length

$$\tau_e = \frac{1}{4\pi\sigma} \sim 10^{-15}\ \mathrm{sec}$$

In this environment, the magnetic field changes slowly. The magnetic decay time is

$$\tau_m \sim \frac{4\pi\sigma a^2}{c^2}$$

For a typical millimeter-sized beam for HEDP, $\tau_m \sim 12ns$, which is much longer than the beam pulse length (~ 1ns). Hence over the pulse duration full charge and current neutralization is a good approximation.

Particle dynamics in adiabatic discharge channel

Since the ions do not see any of the beam space charge, the particle dynamics reduces to single particles in an external B_θ field, given by

$$\frac{d^2x}{dz^2} = -k^2 x$$

where $k^2 = \dfrac{2I}{a^2\left(\beta Mc^3/Ze\right)}$

where x, β, M, and Z are the transverse position, ion speed (normalized to speed of light c), the mass, and charge of the ion respectively, and a discharge current I flows in a channel of radius a. This formula assumes a radially uniform current density. The generalization to non-uniform distribution is straightforward.

The betatron wave number k can vary with z, due to continuous stripping Z (z), as well as channel tapering a (z). The solution to the Hill's Equation

$$\frac{d^2x}{dz^2} = -k^2(z)x$$

is easily obtained in the adiabatic approximation

$$\frac{1}{k^2}\frac{dk}{dz} << 1$$

and is given by

$$\boxed{x = \frac{c}{k^{1/2}} e^{i\int k dz}}$$

In practice, the adiabaticity condition can be met as long as the length of the channel L_c is of the order of the batatron period at entrance $L_i = \frac{2\pi}{k_i}$.

It is important to note that the amplitude of x is proportional to $k^{-1/2}$. As the channel tapers, k increases and x is reduced. The beam envelope, R, which is the ensemble average of the ions, will similarly decrease as it goes down the tapered tube.

Characteristics of beam transport in adiabatic channel

The picture of single particle transport in Z-pinch leads immediately to some general conclusions:

1. The beam envelope within the channel is very insensitive to beam energy spread, since
$$\frac{\delta x}{x} \approx \frac{1}{2}\frac{\delta k}{k} \approx \frac{1}{4}\frac{\delta\gamma}{\gamma} \approx \frac{1}{8}\ \frac{\delta E}{E}.$$

Hence 40 % energy spread leads only to 5% variation in beam envelope.

2. Beam transport is insensitive to spread in ion charge state
$$\frac{\delta x}{x} \sim \frac{1}{4}\ \frac{\delta Z}{Z}$$

3. The reduction of beam size, from initial beam radius R_i to final beam radius R_f, is proportional to the square root of the taper ratio $\frac{a_i}{a_f}$ where *a* is the channel radius

$$\frac{R_i}{R_f} = \left(\frac{a_i}{a_f}\right)^{1/2}$$

4. Finally, the channel can accommodate large emittances with sufficient channel current. Assuming that the beam is in quasi-equilibrium at target, we have

$$\boxed{\frac{\varepsilon_f^2}{R_f^2} = \frac{2I}{\beta Mc^3/Ze}}$$

The final emittance ε_f at target may be slightly higher than the beam emittance ε at entry into the discharge channel, depending on details of the channel current radial distribution. If the distribution is uniform, then the emittance is preserved $\varepsilon_i = \varepsilon_f$

A numerical example

For Ne at $\beta = 0.045$ and Z=7, we can reduce a beam radius from 1mm to 0.5mm if the tapered tube has an initial radius of 2mm and final radius of 0.5mm Assuming beam un-normalized edge emittance of $5\text{x}10^{-5}\pi$ m-rad, the current required is 20kA. The length of the tapered tube is 12 cm.