Accelerator and Final Requirements

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Workshop on Accelerator Driven High Energy Density Physics

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(with many thanks for contributions from Debbie Callahan, Max Tabak, Richard Lee, Dale Welch, Larry Grisham, Richard Briggs, Alex Friedman, Ed Lee, Grant Logan, Jay Marx, Andy Sessler, John Staples, Jonathan Wurtele, Simon Yu)

Basic Requirements

Temperature T > ~ 1 eV to study WDM
Energy Density U ~ 10^{11} - 10^{12} J/m^3
Pressure P ~ 1 - 10 MBar
Strong Coupling Constant Γ > ~ 1

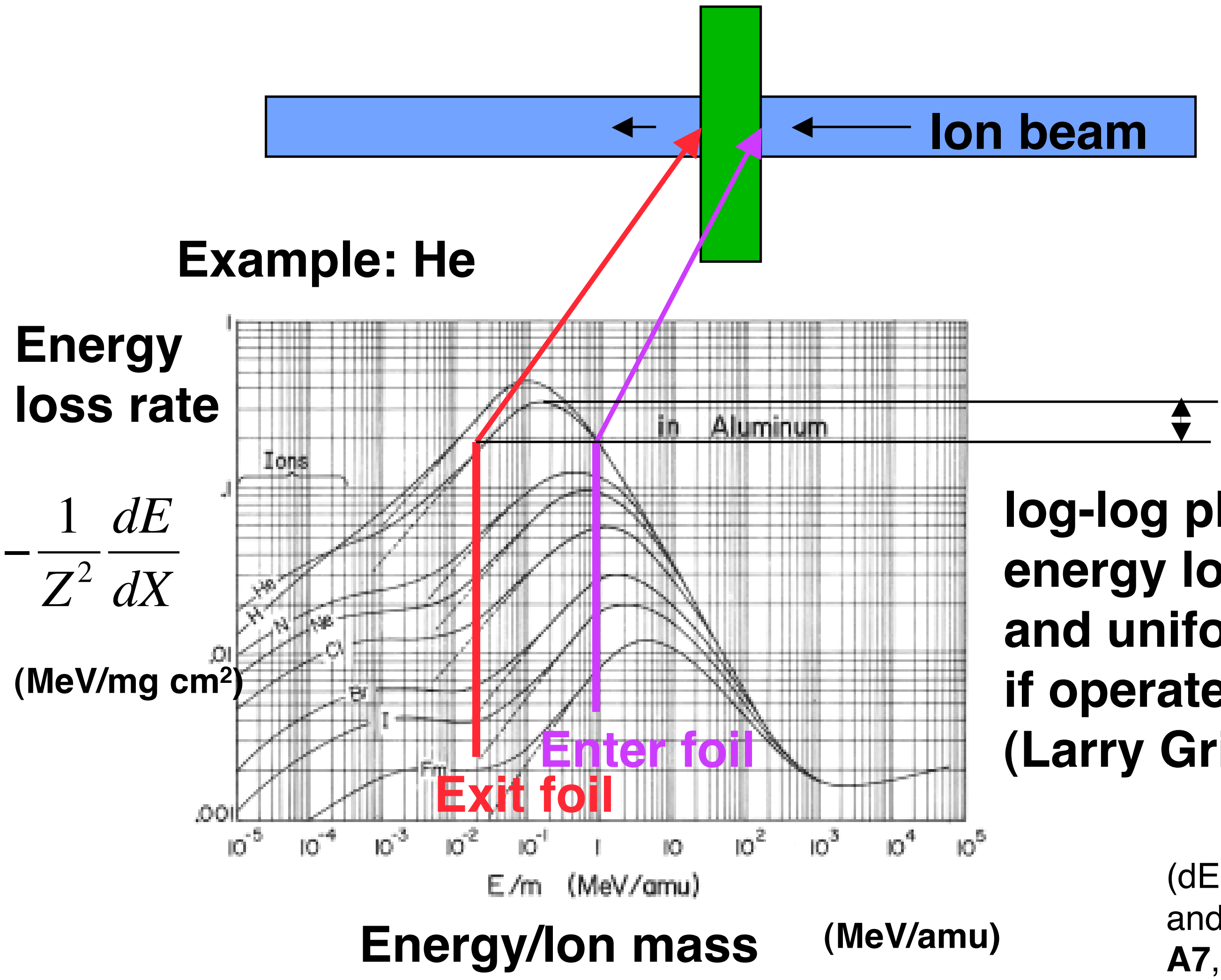
For isochoric heating: Δt must be short enough to avoid cooling from hydrodynamic expansion (to be explained)

Uniformity: $\Delta T/T$ < ~5% (to distinguish various equations of state)

Timescale for building accelerator: ~ 10 years

Strategy: maximize uniformity and the efficient use of beam energy by placing center of foil at Bragg peak

In simplest example, target is a foil of solid or "foam" metal

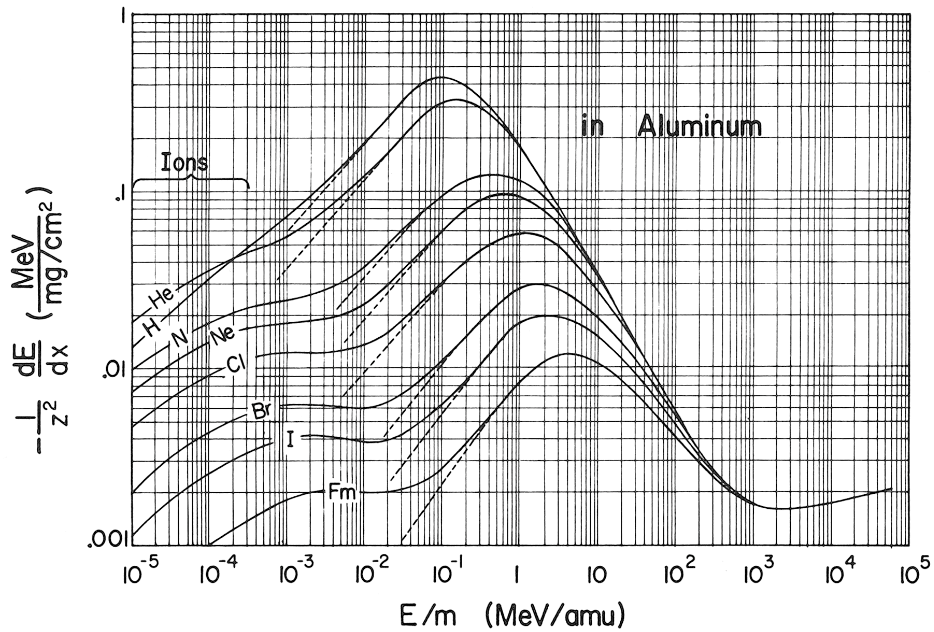


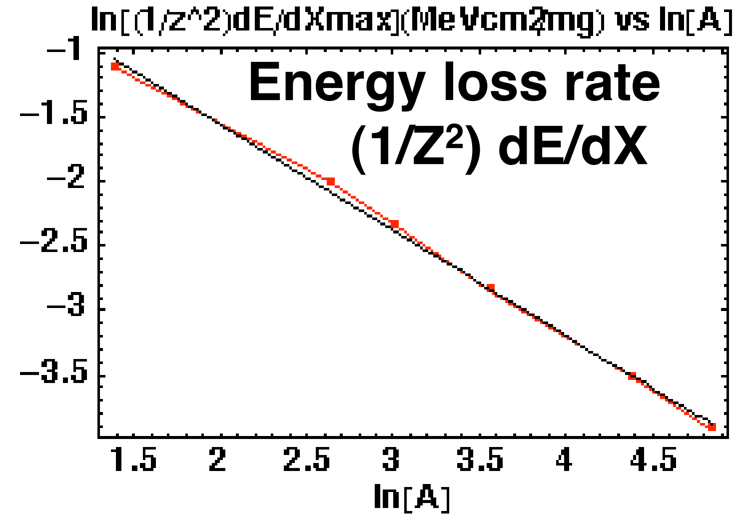
$\Delta dE/dX \propto \Delta T$

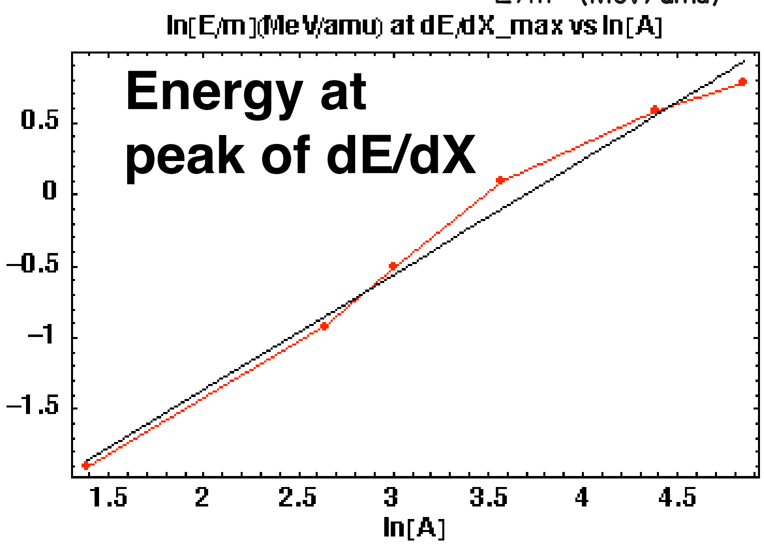
log-log plot => fractional energy loss can be high and uniformity also high if operate at Bragg peak (Larry Grisham, PPPL)

(dEdX figure from L.C Northcliffe and R.F.Schilling, Nuclear Data Tables, **A7**, 233 (1970))

Increasing ion mass, increases energy of Bragg peak, and energy loss rate at Bragg peak







For 4 < A < 126 (He -> I):

Energy at maximum dE/dX:
$E_{dEdXmax} \sim 0.052$ **MeV** $A^{1.803}$

Energy loss rate at maximum dE/dX:
$(1/Z^2)dE/dX_{max} \sim 1.09$ **(MeVcm²/mg)** $A^{-0.82}$
$dE/dX_{max} \sim 0.35$ **(MeVcm²/mg)** $A^{1.07}$

Some scalings

E (at dE/dX_{max}) =~ 0.052 MeV $A^{1.803}$

$\Delta E/E = \sim < 0.50$ **(for a 5% change in dE/dX, half width in energy)**

$Z = 2\Delta E/(\rho\, dE/dX) = \sim (0.55\text{-}0.77)\ \mu\ \ A^{0.733}(\rho_{al}/\rho)$ **(width of foil for 5% change)**

Energy density increases with higher ρ, larger A:

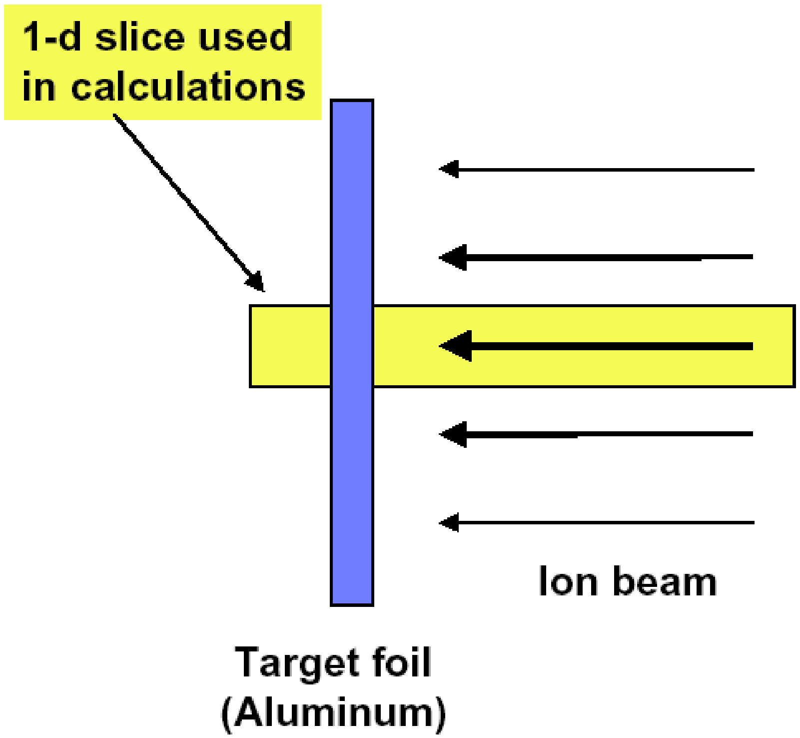
$$U = \frac{N_{ions}E}{\pi r^2 Z} = 3.7\times 10^9 \frac{\mathrm{J}}{\mathrm{m}^3}\left(\frac{N_{ions}}{10^{12}}\right)\left(\frac{1\ \mathrm{mm}}{r}\right)^2\left(\frac{\rho}{\rho_{al}}\right)A^{1.07}$$

Hydro time increases with lower ρ, and weakly on larger A:

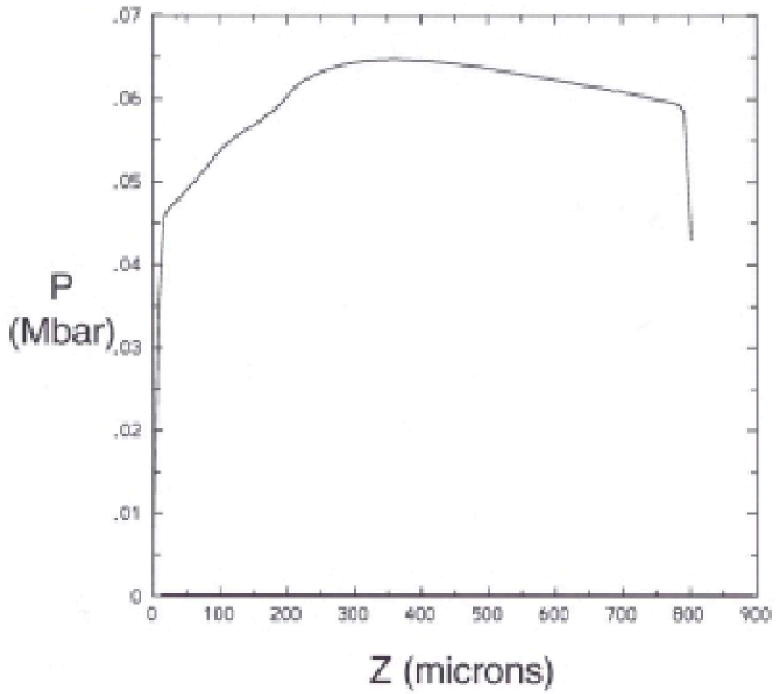
$$t_{hydro} = Z/c_s = \frac{Z}{\sqrt{\gamma(\gamma-1)U/\rho}} = 0.6\times 10^{-9}\mathrm{s}\left(\frac{10^{12}}{N_{ions}}\right)^{1/2}\left(\frac{r}{1\ \mathrm{mm}}\right)\left(\frac{\rho_{al}}{\rho}\right)A^{0.198}$$

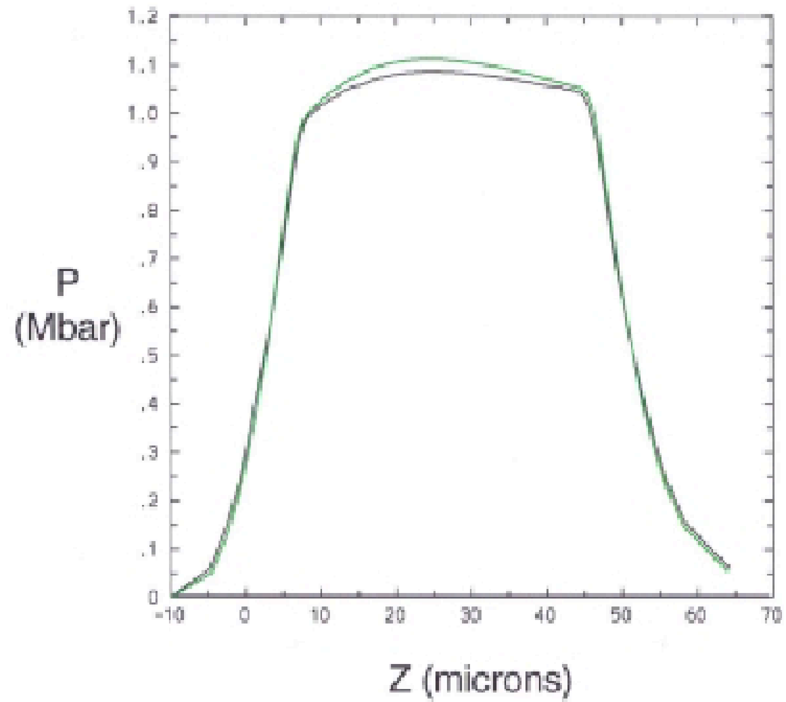
Simulations were carried out by D. Callahan, to explore hydrodynamic effects

- 1-d calculations for the center of the beam
  - Assuming a 1 mm radius Gaussian beam, used 2% of the energy in a 100 micron radius spot
  - 2-d and 3-d effects will make the target expand faster
- "2015" machine
  - Ne^{+1} ion
  - 30 MeV kinetic energy
  - 1 mm radius at best focus
  - 0.5 ns pulse duration
  - 30 J total beam energy
  - 20 - 40 MeV energy spread
  - 60 GW power
  - 3.8 TW/cm^2 center of beam



Using a low density target with the "2015" machine results in more uniformity, but less energy density



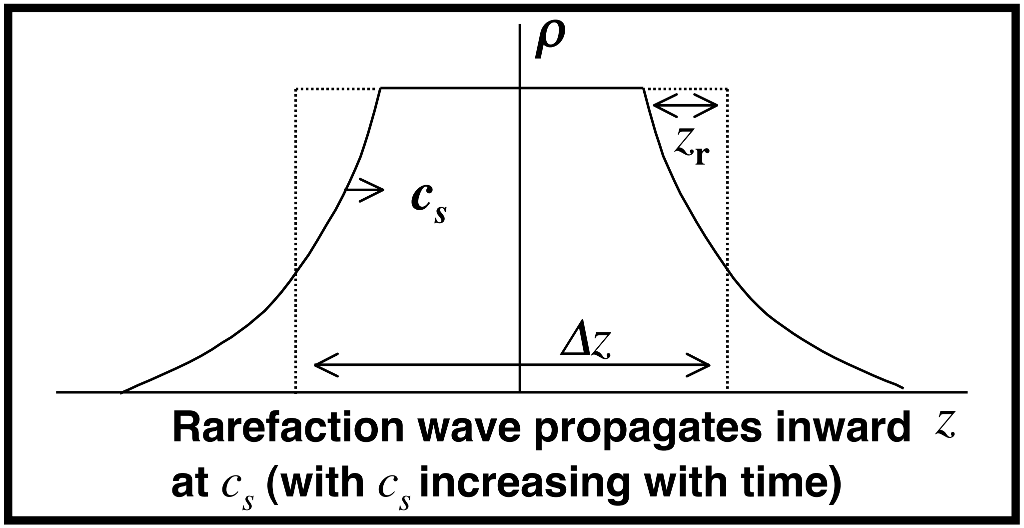


1% solid density
800 microns thick

15% solid density
53 microns thick

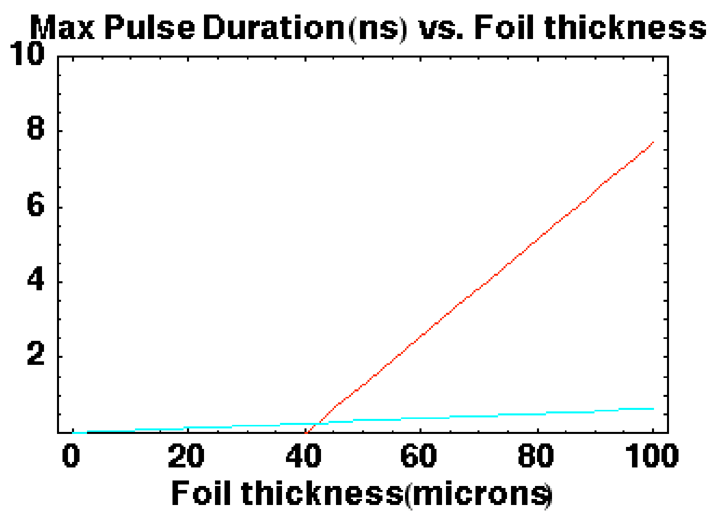
(slide courtesy D. Callahan and M. Tabak, LLNL)

For larger targets ($\Delta z > \Delta z_{min} \sim 40\ \mu$), pulse duration can be significantly longer



Δz_{min} is the minimum length in z for which diagnostics may interrogate the region of interest. We assume $\Delta z_{min} = 40\ \mu$ in this example.

$$\Delta t \le \begin{cases} \dfrac{(\Delta z - \Delta z_{min})}{2(2c_{s*}/3)} & for \quad \Delta z > \Delta z_{min} \\ \dfrac{\Delta z}{2(20)(2c_{s*}/3)} & for \quad \Delta z < \Delta z_{min} \end{cases}$$



Example parameters: Ne^{+1} beam

Ne: Z=10, A=20.17, E_{min}=7.7 MeV, E_{center}=12.1 MeV, E_{max}=20.1 MeV
Δz_{min} = 40 μ

ρ(g/cm^3)(%solid)	0.027 (1%)			0.27 (10%)			2.7 (100%)		
Foil length (μ)	480			48			4.8		
kT (eV)	3.1	4.8	15	4.2	7.3	18	5.9	12	22
Z*	1.1	2.1	2.7	0.56	1.7	2.6	0.56	1.2	2.5
$\Gamma_{ii}=Z^{*2}e^2n_i^{1/3}/kT$	0.45	1.1	0.95	0.30	0.63	1.4	0.30	0.70	1.6
$N_{ions}/(r_{spot}/1mm)^2/10^{12}$	1	3	10	1	3	10	1	3	10
Δt (ns)	84	48	27	3.8	2.2	1.2	0.04	0.03	.014
U (J/m^3)/10^{11}	.015	.045	0.15	0.15	0.45	1.5	1.5	4.5	15

(Eq. of state, Z*: Zeldovich and Raizer model from R.J. Harrach and F. J. Rogers, J. Appl. Phys. **52**, 5592, (1981).)

Example parameters: Cl^{+1} beam

Cl: Z=17, A=35.453, E_{min}=21.1 MeV, E_{center}=48.8 MeV, E_{max}=68.5 MeV
Δz_{min} = 40 μ

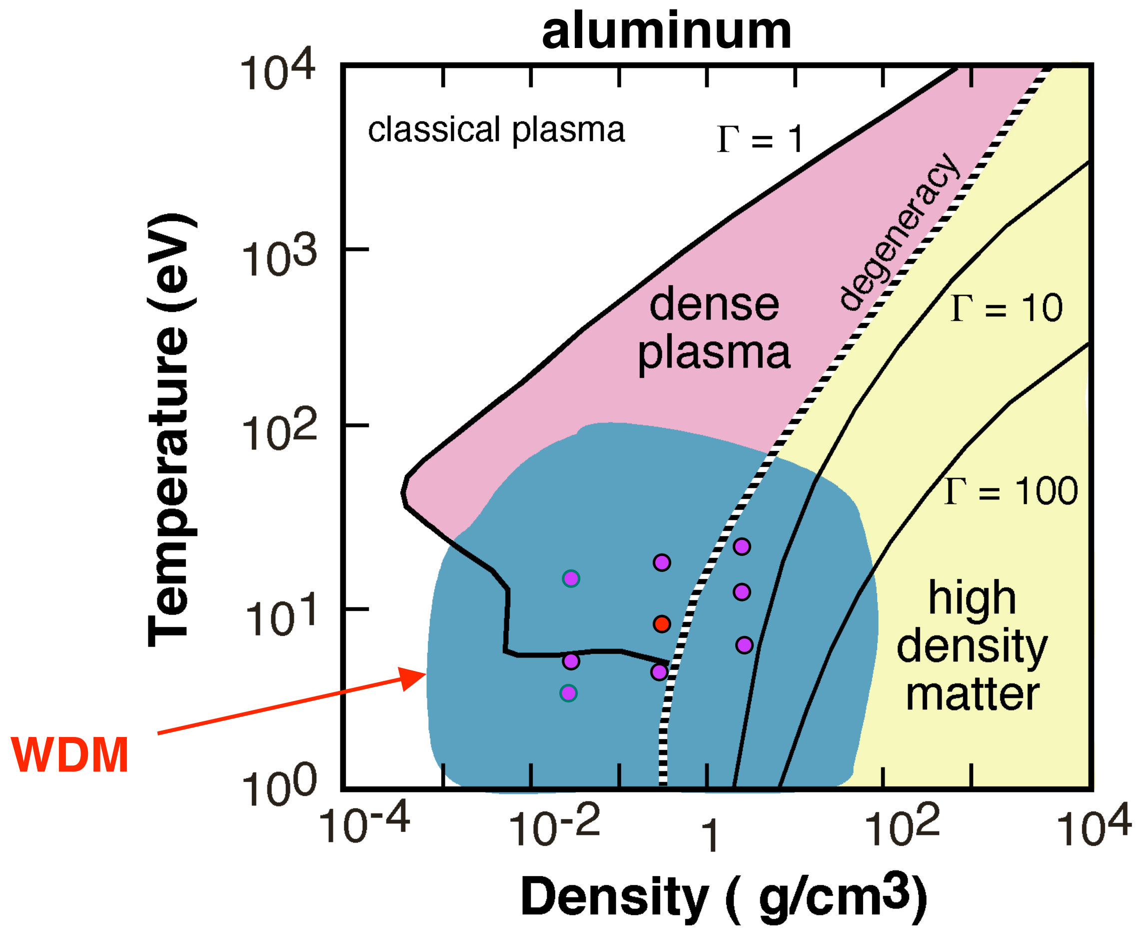
ρ(g/cm^3)(%solid)	0.027 (1%)			0.27 (10%)			2.7 (100%)		
Foil length (μ)	1050			105			10.5		
kT (eV)	3.8	6.5	20	5.2	8.5	25	7.6	14	31
Z*	1.3	2.5	3.5	1.1	2.2	3.2	0.75	1.5	2.8
$\Gamma_{ii}=Z^{*2}e^2n_i^{1/3}/kT$	0.45	1.1	0.71	0.61	1.5	1.1	0.42	0.77	1.5
$N_{ions}/(r_{spot}/1mm)^2/10^{12}$	1	3	10	1	3	10	1	3	10
Δt (ns)	96	56	30	6.2	3.5	2.0	0.050	0.028	.012
U (J/m^3)/10^{11}	.022	.065	0.22	0.22	0.65	2.2	2.2	6.5	22

(Eq. of state, Z*: Zeldovich and Raizer model from R.J. Harrach and F. J. Rogers, J. Appl. Phys. **52**, 5592, (1981).)

Defining the Warm Dense Matter regime

WDM is that region in temperature (T) - density (ρ) space:

1) **Not described as normal condensed matter, i.e., T ~ 0**
2) **Not described by weakly coupled plasma theory**



- Γ is the strong coupling parameter, the ratio of the interaction energy between the particles, V_{ii}, to the kinetic energy, T

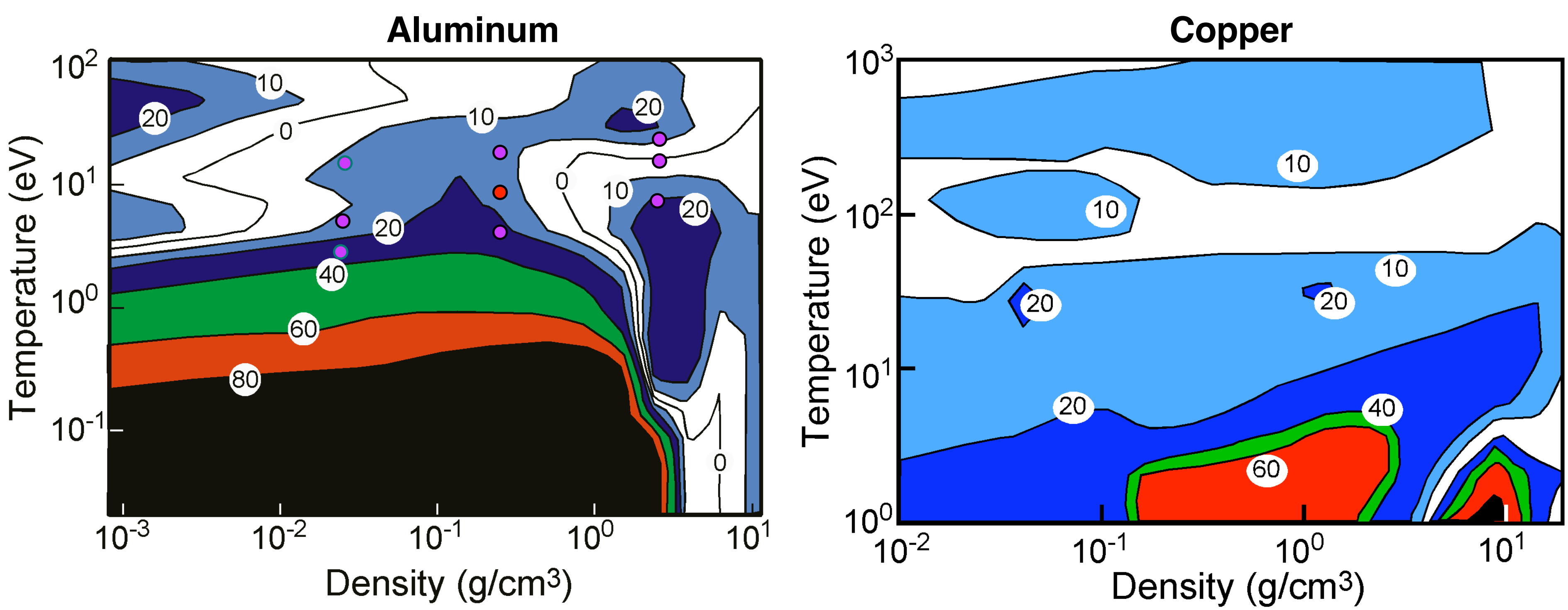
- $$\Gamma = \frac{V_{ii}}{T} = \frac{Z^2 e^2}{r_o T}$$

  where $r_o \propto \frac{1}{\rho^{1/3}}$

(slide courtesy R. Lee, LLNL)

In Warm Dense Matter regime large errors exist even for most studied materials **(slide courtesy R. Lee, LLNL)**

Contours of % differences in pressure



- EOS Differences > 80% are common
- Measurements are *essential* for guidance
- Where there is data the models agree!!
    - Data is along the Hugoniot - single shock ρ-T-P response curve

Accelerator to achieve WDM is challenging -- explores new beam physics regimes

Consider:

20 MeV Ne^+ beam, Δt = 1 ns, N_{ions}=1.0 x 10^{13} particles

Then:

β~ 0.045;

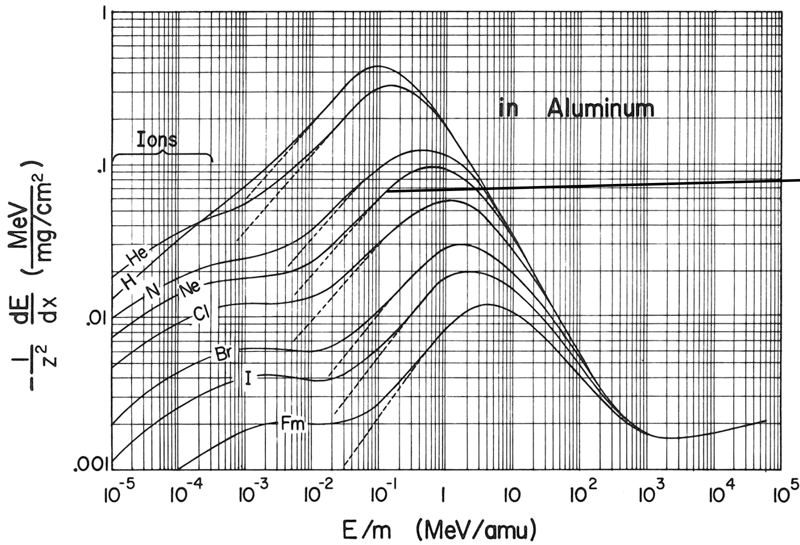
Bunch length $l_b = \beta c\ \Delta t$ = 1.4 cm

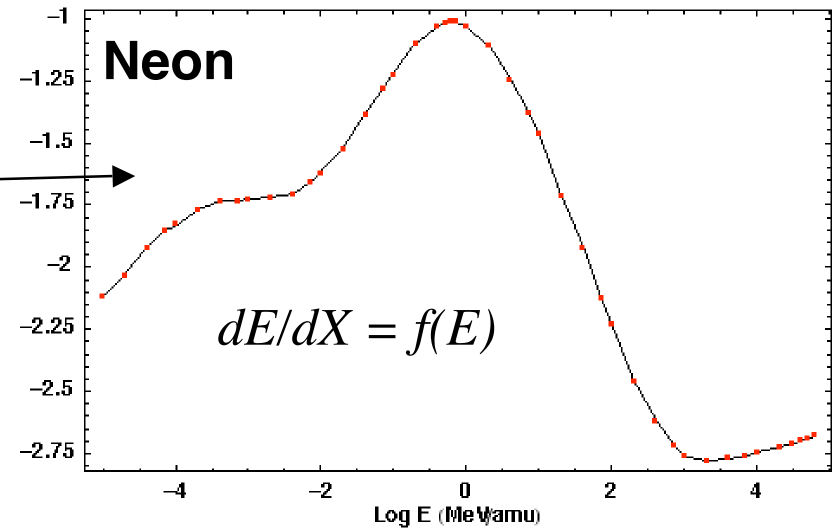
Line charge = $eN_{ions}/\ l_{\mathbf{b}}$=110 μC/m

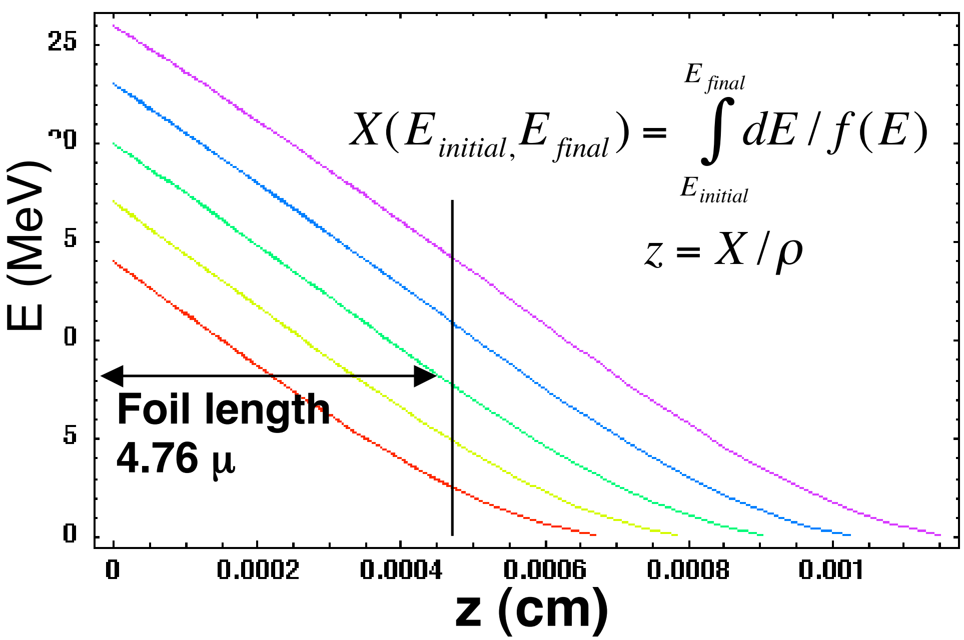
$E_z \sim eN_{ions}/4\pi\varepsilon_0\ l_b^2$ **~ 75 MV/m**

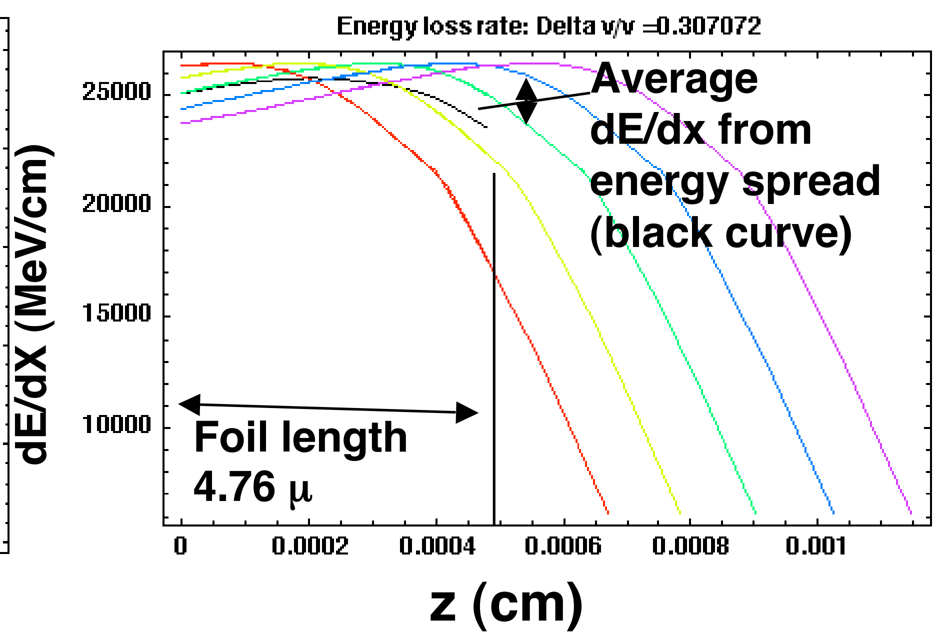
So just to keep beam together requires substantial electric field. (1-2 MV/m typical "limit" in induction linac). So instead: use plasma to neutralize beam during final focus and drift compression

The effect of a velocity spread on temperature uniformity on target can be examined

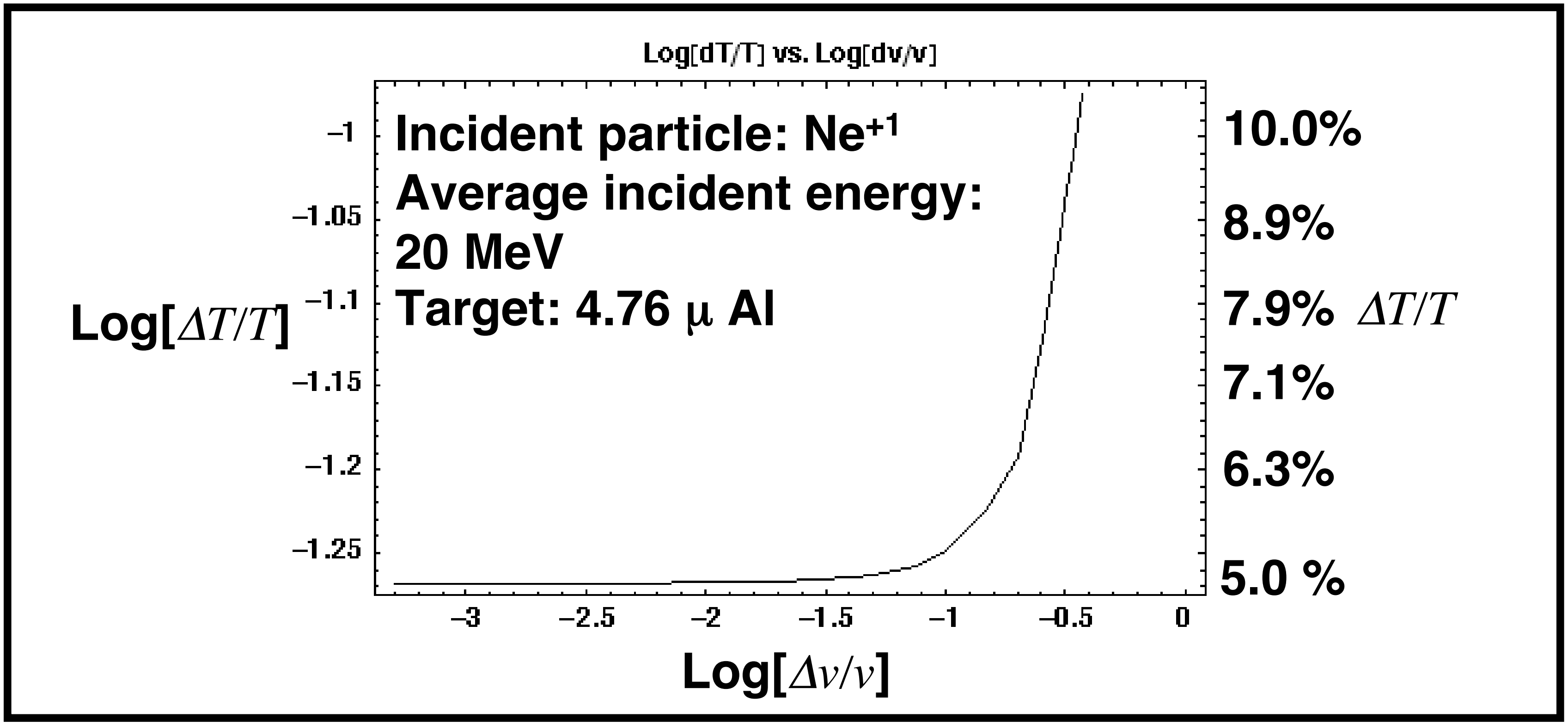








Log ΔT/T vs. Log Δv/v



For a uniform distribution of energies, velocity spread Δv/v does not reduce temperature spread. But as long as velocity spread <~ 10%, temperature spread not significantly increased either.
General result: if ΔE_{spread} <~ $\Delta E_{single\ particle}$ then spread does no harm.

How do you connect the requirements to achievable parameters?

What ion mass, charge, and energy to choose?

1. Source/ Injector

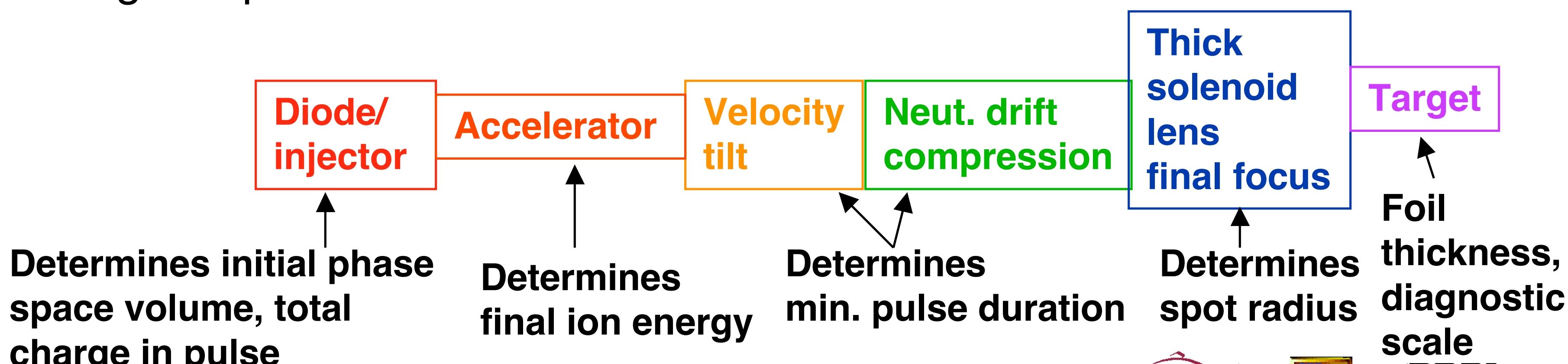
2. Accelerator

3. Drift compression

4. Final focus

5. Target experiment

Consider simple model:



Source/ Injector

a). Beam quality

1. Transverse emittance

$\varepsilon_N = 2r_b(kT/mc^2)^{1/2}$ **=0.051 mm-mrad** $(r_b/0.25\text{cm})\ (20.1/A)^{1/2}\ (kT/2\ \text{eV})^{1/2}$

To avoid voltage breakdown $d = .01\ m\ (V_d/100\ kV)^2\ (100\ kV/V_b)^2$

==> $\varepsilon_f =$ 1.4 mm-mrad $(4/\Delta)\ (kT_s/2\ \text{eV})(V_d/100\ \text{kV})^2)(100\ \text{kV}/V_b)^2(12\ \text{MeV}/qV_f)^{1/2}$

b). Current

$I = (4\pi\varepsilon_0/9)\ (2q/m)^{1/2}\ (V_d^{3/2}/\Delta^2)$ **Here** $\Delta = d/r_b \sim 2.5 - 8 = 4$

$= 0.0745\ A\ (20.1/A/q)^{1/2}\ (4/\Delta)^2\ (V_d/100\ kV)^{3/2}$

-- Total charge $I\Delta t =$ **15** $nC\ (20.1/A/q)^{1/2}\ (4/\Delta)^2\ (V_d/100\ kV)^{3/2}(\Delta t/200ns)$

-- Pulse energy $V_f I\Delta t = 0.18\ J\ (20.1/A/q)^{1/2}\ (4/\Delta)^2\ x$

$(V_d/100\ kV)^{3/2}(\Delta t/200ns)(V_f/12MV)$

Neutralized drift compression allows possibility of very short pulses

For a parabolic pulse the longitudinal envelope equation (including longitudinal thermal spread) for bunch length l is:

$$\frac{d^2 l}{dt^2} = \frac{16\varepsilon_z^2}{l^3} + \frac{4v_0^2 g Q_a l_a}{l^2} \quad \Longrightarrow \quad \left(\frac{\Delta v}{v}\right)_{tilt}^2 = \left(\frac{1}{v}\frac{dl}{dt}\Big|_a\right)^2 = 20\left(\frac{\delta v}{v}\right)_a^2 \left[C^2 - 1\right] + 8gQ_a\left[C - 1\right]$$

Thermal Spread   Space Charge

where $\varepsilon_z^2 \equiv 25\left(\langle \delta v_z^2 \rangle \langle \delta z^2 \rangle - \langle \delta v_z \delta z \rangle^2\right)$

So if velocity spread at end of accelerator $\delta v/v_a \sim 5 \times 10^{-4}$, (corresponding to say an error in voltage $\Delta V/V$ ~0.1% during imposition of velocity tilt),
an initial tilt $\Delta v/v$ <~ 1, and perveance in drift section Q_a = ~0:

$$C_{\max} = \left(\frac{[\Delta v/v]_{tilt}^2}{20[\delta v/v]_a^2} + 1\right)^{1/2} \cong \frac{\Delta v/v_{tilt}}{4.5\ \delta v/v_a}$$

(example: $\Delta v/v_{tilt} = 1$, $\delta v/v_a = 5\text{x}10^{-4}$, ==> $C_{\max} = 450$)

Drift compression and final focus through a thick solenoidal lens

If $$r_{spot}^2 = \frac{4\varepsilon^2 f^2}{\pi^2 r_0^2} + \frac{\pi^2 r_0^2}{4}\left(\frac{\delta p}{p}\right)_t^2$$ then optimum initial beam radius r_{0_opt} which minimizes r_{spot}: $$r_{0\_opt}^2 = \frac{4\varepsilon f}{\pi^2 (\delta p / p)_t}$$

Minimum spot radius at r_{0_opt} **is then:**

$$r_{spot\,\min}^2 = 2\varepsilon f\left(\frac{\delta p}{p}\right)_t$$

At maximum compression $$\left(\frac{\delta p}{p}\right)_t = C\left(\frac{\delta p}{p}\right)_a = \frac{1}{\sqrt{20}}\left(\frac{\Delta v}{v}\right)_{tilt}$$

$$r_{spot\,\min}^2 = \frac{1}{\sqrt{5}}\varepsilon f\left(\frac{\Delta v}{v}\right)_{tilt}$$

Example: for $\Delta v/v_{tilt} = .2$, $\varepsilon = 1.4$ **mm-mrad,** $f=0.5$ **m, ==>** $r_{spotmin} = 0.25$ **mm**

Use: $\varepsilon_N = 2r_b(kT/mc^2)^{1/2}$, $\varepsilon = \varepsilon_N/\beta_f$, *and* $r_b = d/\Delta$, *and* $d = .01\ m\ (V_d/100\ kV)^2\ (100\ kV/V_b)^2$

==> $\varepsilon =$ 1.4 mm-mrad $(4/\Delta)\ (kT_s/2.0\ eV)(V_d/100\ kV)^2(100\ kV/V_b)^2(12\ MeV/qV_f)^{1/2}$

Target requirements for HEDP

Target thickness for 5% temperature variation: $\Delta z = 5.0\mu \left(\frac{A}{20.1}\right)^{0.733}\left(\frac{\rho_{al}}{\rho}\right)$

Final accelerator voltage (at Bragg peak):

$$qV_f(\text{at } dE/dX_{max}) =\sim 0.052 \text{ MeV } A^{1.803}$$

Target energy density U:

$$U = V_f I_d \Delta t_d /(\pi r_{spot}^2 \Delta z)$$

$$= \frac{\overbrace{0.18J\left(\frac{20.1}{A/q}\right)^{1/2}\left(\frac{4}{\Delta}\right)^2\left(\frac{V_d}{100\text{kV}}\right)^{3/2}\left(\frac{\Delta t_d}{200\text{ns}}\right)\left(\frac{V_f}{12\text{MV}}\right)}^{I_d\,\Delta t_d\,V_f}}{\underbrace{\pi(0.25\ \text{mm})^2\left(\frac{f}{0.5\text{m}}\right)\left(\frac{\Delta v/v}{0.2}\right)_{tilt}\left(\frac{4}{\Delta}\right)\left(\frac{V_d}{100\text{kV}}\right)^2\left(\frac{100\text{kV}}{V_b}\right)^2\left(\frac{T_s}{2.0\text{eV}}\right)^{1/2}\left(\frac{12\text{MeV}}{qV_f}\right)^{1/2}}_{r_{spot}^2}\ \underbrace{5.0\mu\left(\frac{A}{20.1}\right)^{0.733}\left(\frac{\rho_{al}}{\rho}\right)}_{\Delta z}}$$

$$U = 1.8\times 10^{11}\,\frac{\text{J}}{\text{m}^3}\left(\frac{2\text{eV}}{\text{kT}_s}\right)^{1/2}\left(\frac{0.2}{\Delta v/v_{tilt}}\right)\left(\frac{q}{1}\right)^{0.32}\left(\frac{4}{\Delta}\right)\left(\frac{\Delta t_d}{200\text{ns}}\right)\left(\frac{V_b}{100\text{kV}}\right)^2\left(\frac{V_d}{100\text{kV}}\right)^{-0.50}\left(\frac{V_f}{12\text{MV}}\right)^{0.815}\left(\frac{0.5\text{m}}{f}\right)\left(\frac{\rho}{\rho_{al}}\right)$$

Timescales

Pulse duration

$$\Delta t_t = \Delta t_d / C = 2.25\text{ns}\left(\frac{\delta v / v_a}{5\times 10^{-4}}\right)\left(\frac{0.2}{\Delta v / v_{tilt}}\right)\left(\frac{\Delta t_d}{200\text{ns}}\right)$$

Hydro expansion ~ $\Delta z/(U/\rho)^{1/2}$

$$t_{hydro} = 0.6\text{ns}\left(\frac{\text{kT}_\text{s}}{2\text{eV}}\right)^{1/4}\left(\frac{\Delta v / v_{tilt}}{0.2}\right)^{1/2}\left(\frac{1}{q}\right)^{0.16}\left(\frac{\Delta}{4}\right)^{1/2}\left(\frac{200\text{ns}}{\Delta t_d}\right)^{1/2}\left(\frac{V_d}{100\text{kV}}\right)^{0.25}\left(\frac{100\text{kV}}{V_b}\right)\left(\frac{f}{1\text{m}}\right)^{1/2}\left(\frac{\rho_{al}}{\rho}\right)$$

Ratio: $t_{hydro}/\Delta t_t$ **must be >> 1**

$$\frac{t_{hydro}}{\Delta t_t} = 0.27\left(\frac{\text{kT}_\text{s}}{2\text{eV}}\right)^{1/4}\left(\frac{5\times 10^{-4}}{\delta v / v_a}\right)\left(\frac{\Delta v / v_{tilt}}{0.2}\right)^{3/2}\left(\frac{20.1}{A}\right)^{0.463}\left(\frac{q}{1}\right)^{0.595}\left(\frac{\Delta}{4}\right)^{1/2}\left(\frac{200\text{ns}}{\Delta t_d}\right)^{3/2}\left(\frac{V_d}{100\text{kV}}\right)^{0.25}\left(\frac{100\text{kV}}{V_b}\right)\left(\frac{f}{1\text{m}}\right)^{1/2}\left(\frac{\rho_{al}}{\rho}\right)$$

Note that $t_{hydro}/\Delta t_t$ **and** U **depend only on ratio of** $\Delta v/v_{tilt}$ / Δt_d**, so lower velocity tilts with smaller diode pulse durations achieve same** $t_{hydro}/\Delta t_t$ **and** U **but with smaller** r_{spot} **to go along with small pulse energy.**

Conclusion

For accelerators to be in an interesting WDM regime, we may consider the following starting point :

For Z=10, (A=20.17), ion energy = 20.1 MeV

$N_{ions}/(r_{spot}/1\ mm)^2 \sim 10^{13}$

$\Delta t < 1.5$ ns

Target temperatures of 18 eV would be reached (in foils 48 μ thick of 10% solid (Al) density) .

During workshop, the accelerator concepts, final focus/drift compression, and experimental working groups must all iterate to obtain self-consistent operating points for a WDM facility.